Risk Sharing and Risk Reduction with Moral Hazard*

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Abstract

This paper studies the design of a Financial Stability Fund under different provisions of incentives. Building a dynamic recursive contract, we generalize the *flexible* moral hazard approach of Georgiadis et al. (2024) in which an agent can freely choose the distribution of shocks next period. Unlike the canonical model of Atkeson and Lucas (1992), this approach provides incentives based on rewarding the agent's increased marginal costs – resulting in increased stochastic dominance – rather than its realized performance. As a result, the optimal contract features *bliss* as opposed to immiseration and the provision of incentives does not disrupt risk-sharing; in fact, risk-reduction is an optimal choice. We propose two ways of bridging the flexible and the canonical approaches by restricting the distribution set in the former and back-loading incentives in the latter, respectively. Building on Ábrahám et al. (2025) calibration of euro area stressed countries, we provide quantitative welfare comparisons of these different moral hazard frameworks: they can depend on how incentive compatibility and limited enforcement constraints interact. These findings offer insights for the design of new official sovereign lending programs.

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JEL classification: E43, E44, E47, E62, F34, F36, F37

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1 Introduction

In models of debt and risk-sharing, moral hazard (MH) is a central concern: a risk-averse borrower can improve its risk profile through effort, but since effort is non-contractible, lenders have to provide *the right incentives for* the borrower to make *the right effort.* Mechanism design, achieves this incorporating incentive compatibility (IC) constraints in the design of constrained efficient debt-and-insurance contracts. However, the introduction of IC constraints is not uniform across theoretical frameworks, leading to different constrained efficient outcomes. Furthermore, while the inclusion of IC constraints in debt and risk-sharing models is well-documented, how these constraints interact with risk-sharing and limited enforcement (LE) constraints remains an open question.

This paper addresses these two issues contrasting the two existing moral hazard theoretical frameworks – the well-established *canonical MH* framework pioneered by Holmstrom (1979) and in dynamic contracts by Atkeson and Lucas (1992), and the new *flexible MH* framework of Georgiadis et al. (2024), which we here extend to dynamic contracts – in the context of debt and risk-sharing contracts with limited enforcement constraints.

In the flexible MH approach, the agent (i.e. the borrower in our economies) can choose any distribution of shocks in a given compact set, each with different costs. The main assumption is that the cost related to the choice of a distribution is Gateau differentiable and monotone in first-order stochastic dominance. The resulting IC ensures that the agent has no incentive to relocate the probability mass across different shock levels. Consequently there is an IC for each shock level which ensures that the marginal cost of shifting the probability mass of a specific shock equates the marginal benefit of doing so.

In contrasts in the canonical MH framework the agent exerts effort to improve – in the first-order stochastic sense – the distribution of shocks. The agent cannot freely choose a distribution since any change in effort has a global effect on the shock distribution. In other words, it is pre-committed to a family of distributions of an observable outcome as function of the effort.[1] The resulting IC ensures the marginal cost of exerting effort equates the associated expected marginal utility gain. There is an IC for each effort level instead of each shock level here. As a result, the flexible and the canonical MH are neither substitutes nor a special case of one another. Nevertheless, the different IC constraints have a basic element in common: first-order stochastic dominance in exercising effort in the canonical approach and in choosing higher marginal cost distributions in the flexible approach.

These two MH frameworks have distinct provisions of incentives. In particular, the canonical

[1] This is similar to a multi-armed bandit problem, where the arms are given and the agent chooses how much effort to exercise in each one.

approach applies the general contracting enforcement principle of 'the carrot or the stick'. In a principal-agent relationship, the provision of incentives rewards the agents when the outcome is good and punishes it otherwise, which in general means that the *ex-post* value of the contract varies accordingly. For instance, the value of the contract decreases for bad outcomes that may occur due to bad luck rather than a lack of effort. When the contract is a risk-sharing agreement between a risk-averse agent and a risk-neutral principal, the IC constraint disrupts the full risk-sharing that could be achieved with observable effort. The risk distribution improves – in stochastic dominance – with effort but cannot be reduced, since all distributions have the same support.[2] As shown by Atkeson and Lucas (1992), in a dynamic context, the disruption increases as a submartingale. That is with an unbounded concave utility the *ex-post* value of the agent decays as a supermartingale to *immiseration*.

In opposition, the flexible MH approach applies the contracting enforcement principle of 'reward the cost beyond the minimum performance' rather than tying incentives to realized outcomes, since outcomes *per se* provide no information when the agent chooses distributions. In fact, the agent may choose to reduce risk if this is not too costly. This distinction has profound implications. The absence of punishment leads to contracts that avoid the aforementioned immiseration effect. In particular, we show that, as long as the agent's LE constraint is not binding and the agent is not too impatient relative to the principal, the *ex-post* value of the agent increases as a submartingale. The contract therefore features *bliss* as opposed to *immiseration*.

We propose two ways of bridging the flexible and the canonical approaches, offering a close basis for comparing various contracts. First, we back-load incentives in the canonical approach based on the following logic. If the LE constraints are not binding in the flexible MH, IC constraints *never* distort risk-sharing (or risk reduction). In opposition, in the canonical MH, IC constraints *always* distort risk sharing. We therefore consider long-term canonical MH contracts as a sequence of subprograms. Within subprograms full-risk sharing is preserved, but when one of the LE constraints binds the subprogram terminates and a new subprogram starts with the initial condition accounting for the performance of the previous sub-contract. That is, in the subprogram IC punishments and rewards are *back-loaded* to the start of the following subprogram. However, since the end of the subprogram is, endogenously, determined by the binding LE constraints, the punishment-reward mechanism must satisfy the constraint. This brings the back-loaded design closer to the flexible MH design. Interestingly, it also brings it closer to existing *official lending programs*, where it is common that the end of a relatively short term program is followed by another program, with a new risk-assessment (i.e. based on the previous performance).

Regarding flexible MH, we restrain the choice of distribution. In other words, the agent is restricted to choose among a specific family of distributions, each with different costs as before but without the possibility to reduce risk. While in the unrestricted flexible MH, when risk-reduction

[2] Otherwise, the principal could elucidate the effort and apply a more severe punishment and reward.

is not too costly, the optimal distribution is a Dirac distribution, it becomes the closest available approximation of a Dirac in the restricted case. This brings the contract closer to the canonical MH design.

Our ultimate interest is in the design of constrained efficient contracts between an impatient and risk-averse sovereign borrower and a patient and risk-neutral financial stability Fund. Therefore, we develop the theory of flexible MH and its comparison with the canonical MH in this broader context. The Fund offers country-specific contracts that guarantee that the sovereign debt is safe and sustainable (i.e. there are no *ex-post* expected losses for the lender). After deriving and characterizing the different Fund contracts, we offer a quantitative exploration using benchmark calibrations – closed to Ábrahám et al. (2025) – for the euro area stressed economies. In comparing different MH frameworks, the economy with the Fund and its outside option economy (an Incomplete Market economy with Defaultable debt: IMD) share the same MH framework. As we show, the effect of the limited enforcement constraints can reverse the welfare rankings. In our simulations the economy with flexible MH is *Pareto dominated* by the other economies with different MH frameworks. This is due to the fact that welfare in the IMD economy is higher with flexible MH, making the outside values in the Fund economy higher and, as a result, the value of the Fund economy lowers. Without LE constraints or equal outside values, the Fund economy with flexible MH is the one Pareto dominating the others.

The paper is organized as follows. Section 2 reviews the literature. Section 3 exposes the environment. Sections 4 and 5 develop the Fund contracts under flexible and canonical MH, respectively. Section 7 contains the quantitative analysis. Section 8 concludes. All proofs are contained in the Appendix.

2 Literature Review

The paper derives the optimal contract between a lender and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994) who considered the case of limited enforcement. The difference with our approach is that we consider two-sided limited enforcement, while the literature has focused on one-sided limited enforcement analyzing the borrower's perspective. We solve the optimal contract by means of the Lagrangian approach of Marcet and Marimon (2019) which has been widely used to account for limited enforcements (e.g. Kehoe and Perri (2002) and Ferrari et al. (2021)) and its combination with moral hazard (e.g. Simpson-Bell (2020) and Ábrahám et al. (2025)).

We develop an optimal contract combining limited enforcement and moral hazard constraints. Our analysis is close to Atkeson (1991) who – similar to Thomas and Worrall (1994) – studies lending contracts in international contexts. However, Atkeson (1991) models moral hazard with respect to consuming or investing the borrowed funds, while we focus on risk-reduction policies. Quadrini (2004) also combine moral hazard and limited enforcement to study when and how contracts are renegotiation-proof. Similarly, Dovis (2019) shows that the combination of moral hazard and limited enforcement can generate a region of *ex post* inefficiency. This is not the focus of our analysis as our contract is both *ex ante* and *ex post* efficient. In addition, Müller et al. (2019) study dynamic sovereign lending contracts with moral hazard, with respect to reform policy efforts, and limited enforcement. Their characterization of the constrained-efficient allocation is more stylised (normal times are an absorbing state) and focuses on one form of moral hazard only.

We contribute to the literature on moral hazard in dynamic macroeconomic models. Our work relates to the pioneer work Prescott and Townsend (1984) who show that, in a static economy with moral hazard and adverse selection problems, a constrained efficient allocation can be the allocation of competitive equilibrium if the space of contracts is restricted to satisfy the corresponding incentive compatibility constraints. We generalize the flexible moral hazard approach of Georgiadis et al. (2024) to a dynamic model. We then contrast the provision of incentives with the canonical dynamic moral hazard model of Atkeson and Lucas (1992). In the canonical model, moral hazard leads to immiseration. This comes from the incentive-compatible mechanism which rewards high types with larger future utility and low types with lower future utility. This mechanism also disrupts risk sharing owing to the lower future utility of low types. We show that the flexible moral hazard approach leads to the opposite of immiseration – what we call *bliss*. In addition, it does not disrupt risk-sharing as the provision of incentives does not rely on the realized outcome. We then propose two ways to minimize the disruption to risk sharing in the canonical model.

Our work more closely contributes to the recent literature on the design of an optimal stability Fund. Roch and Uhlig (2018), Liu et al. (2020) and Callegari et al. (2023) focus on the lender's side of the contract and therefore disregard moral hazard issues. In opposition, Dovis and Kirpalani (2023) account for moral hazard and show that the provision of effort is back-loaded. We build on Ábrahám et al. (2025), where defaultable sovereign debt is transformed into a safe Fund contract, which accounts for moral hazard. They assume that the Fund has an exclusivity contract unlike Liu et al. (2020) and Callegari et al. (2023) who model a Fund which absorbs a minimal amount of debt. We exploit the fact that IC constraints are disruptions to perfect risk-sharing. Our contribution is to provide a more comprehensive analysis of moral hazard in the Fund design.

3 Environment

We introduce flexible moral hazard in the environment studied in Ábrahám et al. (2025). Consider an infinite-horizon small open economy with a single homogenous consumption good in discrete time. A benevolent government acts as a representative agent and takes decisions on behalf of the small open economy.

The economy can produce goods using a decreasing-returns labour technology $y = \theta f(n)$, where $f'(n) > 0$, $f''(n) < 0$, $n \in [0,1]$ denotes labor and θ is a productivity shock. The shock is composed of two parts $\theta \equiv \zeta + g$ where ζ follows a Markov process with compact support $P \subseteq \mathbb{R}^+$ and transition function $\pi^\zeta(\zeta'|\zeta)$. For g, the government can generate any *distribution* with compact support $K \subseteq \mathbb{R}^+$. We define $\mathcal{M}$ as the set of Borel probability measures on K and the lowest as well as the highest possible shock as $\underline{g} = \min\{g \in K\} > 0$ and $\overline{g} = \max\{g \in K\}$, respectively. We denote by $\pi^g(\zeta')$ the distribution of g' conditional on ζ' and $\boldsymbol{\pi}^g = \{\zeta' \in P : \pi^g(\zeta') \in \mathcal{M}\}$ the vector of all such conditional distributions. While the two shocks ζ' and g' are directly contractible, the vector of conditional distributions $\boldsymbol{\pi}^g$ is not. We denote the state at the beginning of a period to be $s \equiv \{\zeta, g\}$.

The government's discounts the future at the rate β, satisfying $\beta < 1/(1+r)$, where r is the risk-free world interest rate. The government's utility is additively separable in consumption, labor and the effort cost of producing. In particular, $U : \mathbb{R}^+ \times [0,1] \times \mathcal{M}^N \to \mathbb{R}$. So, if the government chooses a distribution vector $\boldsymbol{\pi}^g$, supplies labor n and receives consumption c then its payoff is $U(c, n, \boldsymbol{\pi}^g) \equiv u(c) + h(1-n) - \int v(\pi^g(\zeta'))\pi(\zeta'|\zeta)(\mathrm{d}\zeta')$ where $1-n$ denotes leisure.

Regarding consumption and leisure, we assume that preferences take the constant relative risk aversion form. For the production distribution, we assume that the cost of producing is continuous, convex, Gateaux twice differentiable and that producing more costs more. We also normalize the Gateaux derivative to be zero at $\underline{g}$.

Assumption 1 (Monotonicity, Differentiability and Convexity) *The utility functions from consumption and leisure take the constant relative risk aversion form* $u(c) + h(1-n) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + \psi\frac{(1-n)^{1-\sigma_n}}{1-\sigma_n}$ *with* $\sigma_c \in (0,1)$, $\sigma_n \geq 0$ *and* $\psi \geq 0$. *The utility function from effort,* $v : \mathcal{M} \to \mathbb{R}$, *is continuous and convex. Moreover, it is Gateaux twice differentiable where* $v_{\pi^g} : K \to \mathbb{R}^+$ *and* $w_{\pi^g} : K \times K \to \mathbb{R}^+$ *denote the first and the second Gateaux derivative, respectively. We normalize* $v_{\pi^g}(\underline{g}) = 0$. *Finally, if the distribution* π^g *first-order stochastically dominates* $\tilde{\pi}^g$ *then* $v(\pi^g) \geq v(\tilde{\pi}^g)$.

As the cost related to the choice of distribution is a key variable in the analysis, we give an example of the cost function that we later use in the quantitative section. Following Georgiadis et al. (2024), let $K : \mathbb{R} \to \mathbb{R}$ be an increasing, convex and differentiable function and $v(\pi^g) = K[\int(g - \underline{g})\pi^g(\mathrm{d}g)]$. The marginal cost is then given by $v_{\pi^g}(g) = K'[\int(y - \underline{g})\pi^g(\mathrm{d}y)](g - \underline{g})$.

The borrowing country can manage its private and public debt liabilities with the help of a Financial Stability Fund (Fund), which acts as a benevolent risk-neutral planner who has access to the international capital markets at the risk-free rate.

The country can default on its liabilities. We assume that the country's outside option corresponds to the autarky value of the standard incomplete market model with default which is

$$V^D(s) = \max_{n,\boldsymbol{\pi}^g} \left\{ U(\theta^d f(n), n, \boldsymbol{\pi}^g) + \beta \int \int \left[(1-\lambda)V^D(s') + \lambda J(s',0)\right] \pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta') \right\}, \tag{1}$$

where $\theta^d \leq \theta$ contains the penalty for defaulting and $\lambda \geq 0$ is the probability to re-access the private bond market. Furthermore, V^D corresponds to the value under financial autarky and J to the value of reintegrating the private bond market without the Fund. More precisely, $J(s,b) = \max_{D\in\{0,1\}}\{(1-D)V^P(s,b) + DV^D(s)\}$, with

$$\begin{aligned} V^P(s,b) = & \max_{\{c,n,\boldsymbol{\pi}^g,b'\}} \left\{ U(c,n,\boldsymbol{\pi}^g) + \beta \int \int \left[J(s',b')\right] \pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta') \right\} \\ & \text{s.t. } c + q(s,b')(b' - \delta b) \leq \theta f(n) + (1-\delta+\delta\kappa)b. \end{aligned}$$

In the private bond market, the government can borrow long-term defaultable bonds, b', at a unit price of $q_p(s,b')$. A fraction $1-\delta$ of each bond matures today and the remaining fraction δ is rolled-over and pays a coupon κ. Private lenders are competitive and the price of one unit of private bond is given by $q_p(s,b') = \frac{1}{1+1}\int\int(1-D(s',b'))[1-\delta+\delta\kappa+\delta q_p(s',b'')]\pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta')$ where D is the default policy taking value one in case of default and zero otherwise.

We end this section by providing more structure to the underlying stochastic structure.

Definition 1 (Partial-order in s) *We say that $s \geq \tilde{s}$ if, and only if, $\zeta \geq \tilde{\zeta}$ and the distribution π^g first-order stochastically dominates $\tilde{\pi}^g$.*

The following monotonicity assumption on the outside values, guarantees that they are well-behaved with respect to the exogenous shock ζ and the endogenous distribution of g, which in turn will allow us to make meaningful comparisons of different moral hazard framework.

Assumption 2 (Monotonicity of $V^D(s)$ with respect to the s partial-order) *If $s \geq \tilde{s}$ then $V^D(s) \geq V^D(\tilde{s})$, with inequality if, in addition, $s \neq \tilde{s}$.*

Lemma 1 *If, in addition to $s \geq \tilde{s}$, $\int \zeta'\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta') \geq \int \zeta'\pi^\zeta(\zeta'|\tilde{\zeta})(\mathrm{d}\zeta')$, then Assumption 2 is satisfied.*

Note that the additional condition guarantees that higher ζ shocks are associated with relatively higher expected ζ' shocks (which is compatible with mean-reversion), a property that stochastic dominance guarantees for the $\boldsymbol{\pi}^g$ process.

4 The Fund under Flexible Moral Hazard

The Fund contract chooses a state-contingent sequence of consumption, labor and shock distribution that maximises the life life-time utility of the borrower given some initial level of the borrower's debt. It seeks to provide risk-sharing between the contracting parties. However, limited enforcement and moral hazard frictions preclude perfect risk-sharing.

The optimal contract is self-enforcing through the presence of two limited-enforcement constraints. First, we assume that if the country ever defaults on the Fund contract, it will not be able to sign a new contract with the Fund and will enter the markets for defaultable long-term debt as a defaulter. The Fund contract, however, makes sure that the country never finds it optimal to renege the contract. Second, the contract also prevents the Fund from ever incurring undesired expected losses, i.e. undesired permanent transfers.

In addition, the contract also has an incentive compatibility constraint, since the distribution of g is non-contractible (i.e. it is private information, or a sovereign right of the country). Thus, the long term contract must provide sufficient incentives for the country to implement a constrained efficient distribution.

4.1 The Constraints

Given the aforementioned frictions, the Fund has to account for three differents constraints. The first one is the *limited enforcement constraint* for the borrower. For any $s^t, t \geq 0$, it should be that

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty} \beta^{j-t} U(c(s^j), n(s^j), \boldsymbol{\pi}^g_{j+1})\right] \geq V^D(s_t). \tag{2}$$

The outside value for the borrower is denoted by $V^D(s_t)$ and is given by (1). For now, we just need to assume that $V^D(s)$ is bounded below, increasing in s, and that there is always room for risk-sharing, in spite of the limited enforcement constraints.[3] In principle, a diverse set of default scenarios can satisfy these requirements. Note also that the notation is implicit about the fact that expectations are conditional on the implemented distribution sequence and ζ_t.

The second constraint is the *limited enforcement constraint* for the lender. For any $s^t, t \geq 0$, it should hold that

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \left(\theta_j f(n(s^j)) - c(s^j)\right)\right] \geq Z(s_t). \tag{3}$$

The finite outside option of the lender $Z(s_t) \leq 0$ measures the extent of *ex-post* redistribution the Fund is willing to tolerate. That is, if $Z(s_t) < 0$ the Fund is allowed to make a permanent

[3] In particular, $V^D(\underline{s}) < V^D(\overline{s})$.

loss in terms of lifetime expected net present value – i.e. the Fund can find better investment opportunities in the international financial market and if it does not renege it is because it has committed to sustaining $Z(s_t) < 0$. Clearly, the level of $Z(s_t)$ has an important impact on the amount of risk sharing in our environment and it can thus be interpreted as the extent of solidarity the Fund is willing to accept in state s, as in Tirole (2015).

Finally, the last constraint is the *incentive compatibility constraint* (ICC). Define $V^b(s^t) = \mathbb{E}_t[\sum_{j=0}^{\infty} \beta^j U(c(s^{t+j}), n(s^{t+j}), \boldsymbol{\pi}^g_{t+j+1})]$ as the value of the borrower at time t. For any $s^t, t \geq 0$ and a given consumption and leisure schedules $\{c(s^t), n(s^t)\}_{t=0}^{\infty}$, the optimal vector of distributions is

$$\boldsymbol{\pi}^g_{t+1} = \underset{\tilde{\boldsymbol{\pi}}^g}{\operatorname{argmax}} \left\{ -\int v(\tilde{\pi}^g(\zeta^{t+1}))\pi^\zeta(\zeta^{t+1}|\zeta^t)(\mathrm{d}\zeta^{t+1}) + \beta \int\int V^b(s^{t+1})\tilde{\pi}^g(\zeta^{t+1})(\mathrm{d}g^{t+1})\pi^\zeta(\zeta^{t+1}|\zeta^t)(\mathrm{d}\zeta^{t+1}) \right\},$$

which given Assumption 1 can be rewritten as

$$\boldsymbol{\pi}^g_{t+1} = \underset{\tilde{\boldsymbol{\pi}}^g}{\operatorname{argmax}} \left\{ \int\int \left[\beta V^b(s^{t+1}) - v_{\pi^g_{t+1}}(g^{t+1})\right] \tilde{\pi}^g(\zeta^{t+1})(\mathrm{d}g^{t+1})\pi^\zeta(\zeta^{t+1}|\zeta^t)(\mathrm{d}\zeta^{t+1}) \right\}.$$

We then re-scale the maximization problem by stating the gain and cost of effort in relative terms to the *no-effort option*,

$$\boldsymbol{\pi}^g_{t+1} = \underset{\tilde{\boldsymbol{\pi}}^g}{\operatorname{argmax}} \Bigg\{ \int\int \left[\beta \left(V^b(s^{t+1}) - V^b_0(\zeta^{t+1})\right) - \left(v_{\pi^g_{t+1}}(g^{t+1}) - v_{\pi^g_{t+1}}(\underline{g})\right)\right] \tilde{\pi}^g(\zeta^{t+1})(\mathrm{d}g^{t+1})\pi^\zeta(\zeta^{t+1}|\zeta^t)(\mathrm{d}\zeta^{t+}$$

where $V^b_0(\zeta^{t+1}) = V^b(\{\zeta^{t+1}, \underline{g}\})$ and $v_{\pi^g_{t+1}}(\underline{g}) = 0$ by Assumption 1. Therefore the ICC is for any $s^{t+1}, t \geq 0$,

$$v_{\pi^g_{t+1}(\zeta^{t+1})}(g^{t+1}) = \beta \left(V^b(s^{t+1}) - V^b_0(\zeta^{t+1})\right), \tag{4}$$

Note that $v_{\pi^g_{t+1}(\zeta^{t+1})}(g^{t+1})$ is a Gateaux derivative which gives the derivative of the cost of production at the specific distribution $\pi^g_{t+1}(\zeta^{t+1})$.

By defining the ICC in this way, we use the first-order approach. That is we replace the agent's full optimization problem with respect to $\tilde{\boldsymbol{\pi}}^g$ by its necessary first-order condition. Our approach builds on the Gateau differentiability as in Georgiadis et al. (2024) which is more general than the standard approach of Rogerson (1985). The government chooses directly a distribution and not simply a level of effort for a given distribution. That is the government can arbitrarily manipulate the relative likelihood of any collection of shocks. As a result, there is no information in the realization of any g. The provision of incentive is therefore solely directed towards the compensation of the marginal cost of producing each shock. The next sections compare the outcome of the two approaches in more details.

4.2 The Long Term Contract

In its extensive form, the *Fund contract* specifies that in state $s^t = (s_0, \ldots, s_t)$, the country consumes $c(s^t)$, uses labour $n(s^t)$ and chooses the distribution vector $\boldsymbol{\pi}^g_{t+1}$, resulting in a transfer to the Fund of $\theta f(n(s^t)) - c(s^t)$, implying that the country is effectively borrowing. With *two-sided limited enforcement* and *moral hazard*, an optimal Fund contract is a solution to the following Fund problem:

$$\max_{\{c(s^t),n(s^t),\boldsymbol{\pi}^g_{t+1}\}} \mathbb{E}_0 \left[\alpha_{b,0} \sum_{t=0}^{\infty} \beta^t U(c(s^t), n(s^t), \boldsymbol{\pi}^g_{t+1}) + \alpha_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[\theta_t f(n(s^t)) - c(s^t) \right] \right]$$
$$\text{s.t.} \quad (2), \ (3), \ \text{and} \ (4), \ \forall s^t, t \geq 0.$$

Note that $(\alpha_{b,0}, \alpha_{l,0})$ are the initial Pareto weights, which are key for our interpretation of the Fund contract as a lending contract. In particular, we show later that the initial relative Pareto weight determines uniquely the level of debt that the Fund takes over when the country joins.

Given (2), (3) and (4), we take the following interiority assumption to ensure the uniform boundedness of the Lagrange multipliers.

Assumption 3 (Interiority) *There is an $\epsilon > 0$, such that, for all $s_0 \in S$ there is a program $\{\tilde{c}(s^t), \tilde{n}(s^t), \tilde{\boldsymbol{\pi}^g}_t\}_{t=0}^{\infty}$ satisfying constraints (2) and (3) when, on the right-hand side, $V^D(s_t)$ and $Z(s_t)$ are replaced by $V^D(s_t) + \epsilon$ and $Z(s_t) + \epsilon$, respectively, and similarly, when in (4) $\overline{v}_{\tilde{\pi}^g_{t+1}(\zeta^{t+1})}(g^{t+1})$ is replaced by $\overline{v}_{\tilde{\pi}^g_{t+1}(\zeta^{t+1})}(g^{t+1}) + \epsilon$ and $=$ is replaced by $\leq$.*

For constraints (2) and (3), this assumption requires that, in spite of the *enforcement constraints*, there are strictly positive rents to be shared since otherwise there may not be a constrained-efficient risk-sharing contract. The last part of this assumption is satisfied if an interior distribution vector $\boldsymbol{\pi}^g_{t+1}$ is feasible.[4]

Following Marcet and Marimon (2019) and Mele (2011), we can rewrite the Fund contract

[4] Note that these assumption can easily be satisfied since there are potential positive gains from risk-sharing (and borrowing an lending) in a contract between a risk-averse agent and a risk-neutral agent as long as there is a sufficiently high penalty for default. In other words, for β high enough, the risk sharing gains are strictly positive if $Z = 0$. The interiority of the distribution can be guaranteed if full risk sharing is not the only feasible allocation and appropriate Inada conditions are imposed on the cost $v(\pi^g)$.

problem as a saddle-point Lagrangian problem:

$$
\begin{aligned}
\text{SP} \min_{\{\gamma_b(s^t),\gamma_l(s^t),\boldsymbol{\xi}(s^{t+1})\}} \max_{\{c(s^t),n(s^t),\boldsymbol{\pi}^g_{t+1}\}} & \Bigg\{ \mathbb{E}_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \Big(\alpha_{b,t}(s^t) U\left(c(s^t), n(s^t), \boldsymbol{\pi}^g_{t+1}\right) \\
& - \xi(s^{t+1}) \Big(\beta V_0^b(\zeta^{t+1}) + v_{\pi^g_{t+1}(\zeta^{t+1})}(g^{t+1}) \Big) \\
& + \gamma_b(s^t) \big[U(c(s^t), n(s^t), \boldsymbol{\pi}^g_{t+1}) - V^D(s_t) \big] \Big) \\
& + \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \Big(\alpha_{l,t+1}(s^t) c_l(s^t) - \gamma_l(s^t) \big[c_l(s^t) - Z(s^t) \big] \Big) \Bigg] \Bigg\} \\
\text{s.t.} \qquad & c_l(s^t) = \theta_t f(n(s^t)) - c(s^t) \\
& \alpha_{b,t+1}(s^{t+1}) = \alpha_{b,t}(s^t) + \gamma_b(s^t) + \xi(s^{t+1}), \\
& \alpha_{l,t+1}(s^t) = \alpha_{l,t}(s^t) + \gamma_l(s^t), \\
& \alpha_{b,0}(s^0) \equiv \alpha_{b,0}, \alpha_{l,0}(s^0) \equiv \alpha_{l,0} \text{ given,}
\end{aligned}
$$

where $\gamma_b(s^t)$, $\gamma_l(s^t)$ and $\xi(s^{t+1})$ are the Lagrange multipliers of the limited enforcement constraints in (2) and in (3), and the ICC in (4), respectively, in state s^{t+1}. The vector of multipliers attached to the ICC is then given by $\boldsymbol{\xi}(s^{t+1}) = \{\zeta^{t+1} \in P : \xi(\{\zeta^{t+1}, g^{t+1}\})\}$.

The above formulation of the problem defines two new co-state variables $\alpha_b(s^t)$ and $\alpha_l(s^t)$, which represent the temporary Pareto weights of the borrower and the lender respectively. These variables are initialized at the original Pareto weights and become time-variant because of the limited commitment and moral hazard frictions. In particular, a binding LE constraint of the borrower (lender) will imply a higher welfare weight of the borrower (lender) so that it does not leave the contract. In addition, the moral hazard friction implies that the co-state variable of the borrower will increase as $\xi(s^{t+1}) \geq 0$.

Given the homogeneity of degree one of the maximization problem in $(\alpha_{b,t}, \alpha_{l,t})$, only relative Pareto weights, defined as $x_t(s^t) \equiv \alpha_{l,t}(s^t)/\alpha_{b,t}(s^t)$, matter for the allocations, and this allows us to reduce the dimensionality of the co-state vector and write the problem recursively by using a convenient normalization. Let $\eta \equiv \beta(1+r) < 1$. We normalize the multipliers as follows:

$$
\nu_b(s^t) = \frac{\gamma_b(s^t)}{\alpha_{b,t}(s^t)}, \; \nu_l(s^t) = \frac{\gamma_l(s^t)}{\alpha_{l,t}(s^t)} \text{ and } \varrho(s^{t+1}) = \frac{\xi(s^{t+1})}{\alpha_{b,t}(s^t)}. \tag{5}
$$

The *Saddle-Point Functional Equation* (**SPFE**) — i.e. the saddle-point version of Bellman's equation — is given by

$$
\begin{aligned}
FV(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\boldsymbol{\varrho}'\}} \max_{\{c,n,\boldsymbol{\pi}^g\}} & \Big\{ x \Big[(1+\nu_b) U(c,n,\boldsymbol{\pi}^\zeta) - \nu_b V^D(s) \Big] \qquad (6) \\
& + \Big[(1+\nu_l)[\theta f(n) - c] - \nu_l Z(s) \Big] \\
& + \int \int \left[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - x \varrho(s') \Big(v_{\pi^g(\zeta')}(g') + V_0^b(x',\zeta') \Big) \right] \pi^g(\zeta')(\mathrm{d}g') \pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta') \Big\} \\
\text{s.t.} \quad & x'(s') \equiv \overline{x}'(s) + \hat{x}'(s') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho'(s')}{1+\nu_l} \right] \eta x, \qquad (7)
\end{aligned}
$$

where $\boldsymbol{\varrho}'(s') \equiv \{\varrho_{\zeta_1}(s'), \ldots, \varrho_{\zeta_N}(s')\}$ corresponds to the vector of multipliers attached to the ICC. The Fund's value functions can be decomposed as follows

$$FV(x,s) = xV^b(x,s) + V^l(x,s) \text{ with} \tag{8}$$

$$V^l(x,s) = \theta f(n) - c + \frac{1}{1+r}\int\int \left[V^l(x'(s'),s')\right]\pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta'), \tag{9}$$

$$V^b(x,s) = U(c,n,\boldsymbol{\pi}^g) + \beta\int\int \left[V^b(x'(s'),s')\right]\pi^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta') \tag{10}$$

The policy functions for consumption and labor of the Fund contract must solve the first-order conditions of the SPFE. In particular, $c(x,s)$ and $n(x,s)$ satisfy

$$u'(c(x,s)) = \frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x}, \tag{11}$$

$$\frac{h'(1-n(x,s))}{u'(c(x,s))} = \theta f'(n(x,s)). \tag{12}$$

These conditions are standard. The borrower's consumption is determined by its endogenous relative Pareto weight and, given that preferences are separable, the labor supply is not distorted. Turning to the optimal distribution, note that, given $\{\nu_b, \nu_l, \boldsymbol{\varrho}'\}$ and $\{c,n\}$, $\boldsymbol{\pi^g}$ maximizes $FV(x,s)$ if it maximizes

$$\int\int\left[\frac{1+\nu_l}{1+r}\tilde{FV}(x'(s'),s') - x(1+\nu_b)v_{\pi^g(\zeta')}(g') - x\varrho(s')v_{\tilde{\pi}^g(\zeta')}(g')\right]\tilde{\pi}^g(\zeta')(\mathrm{d}g')\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta')$$

where $\tilde{FV}(x,s) = x\left[V^b(x,s) - V^b_0(x,\zeta)\right] + \left[V^l(x,s) - V^l_0(x,\zeta)\right]$ with $V^b_0(x,\zeta) = V^b(x,\{\zeta,\underline{g}\})$ and $V^l_0(x,\zeta) = V^l(x,\{\zeta,\underline{g}\})$. Given this, the first-order condition can be expressed as $\Pi_{\pi^g(\zeta')}(s') = 0$, where

$$\begin{aligned}\Pi_{\pi^g(\zeta')}(s') \equiv\, & x\left((1+\nu_b)+\varrho(s')\right)\left[\beta\left(V^b(x',s') - V^b_0(x',\zeta')\right) - v_{\pi^g(\zeta')}(g')\right] \\ &+ \frac{1+\nu_l}{1+r}\left[V^l(x',s') - V^l_0(x',\zeta')\right] - x\int\varrho(\zeta',z)w_{\pi^g(\zeta')}(z,g')\pi^g(\zeta')(\mathrm{d}z),\end{aligned}$$

The second derivative of v, $w_{\pi^g}(z,g')$, represents the change in the marginal cost of generating shock z associated with a slight increase in the probability of shock g'.[5] Given that $v_{\pi^g(\zeta')}(g') = \beta[V^b(s') - V^b_0(x',\zeta')]$ from the ICC, the optimal distribution is such that

$$\frac{1}{1+r}\left[V^l(x',s') - V^l_0(x',\zeta')\right] = xQ_{\pi^g(\zeta')}(g') \tag{13}$$

where $Q_{\pi^g(\zeta')}(g') = \int\tilde{\varrho}(\zeta',z)w_{\pi^g(\zeta')}(z,g')\pi^g(\zeta')(\mathrm{d}z)$ and $\tilde{\varrho}(\zeta',z) = \frac{\varrho(\zeta',z)}{1+\nu_l}$.

The following proposition characterize the optimal distribution. Before this, define $\delta_{g'(\zeta')}$ as the Dirac distribution generating shock $g'(\zeta')$ with probability 1.

[5] We assume that this second derivative exists (and it does in our quantitative analysis); see, the discussion of Assumption 3 in Georgiadis et al. (2024).

Proposition 1 (Optimal Distribution) *If $w_{\pi^g(\zeta')}(g')$ is constant for every g' and $\pi^g(\zeta')$, then $\Pi_{\pi^g(\zeta')}$ is strictly quasiconcave for every $\pi^g(\zeta')$. The optimal distribution has only one g'. The Fund's problem is thus given by*

$$FV(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\boldsymbol{\varrho}\}} \max_{\{c,n,\boldsymbol{g}'\}} \Big\{ x\Big[(1+\nu_b)U(c,n,\boldsymbol{\delta}) - \nu_b V^D(s)\Big] + \Big[(1+\nu_l)[\theta f(n) - c] - \nu_l Z(s)\Big]$$
$$+ \int \Big[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - x\varrho_{\delta_{g'(\zeta')}}(g'(\zeta'))\Big(v_{\delta_{g'(\zeta')}}(g'(\zeta')) + V_0^b(x',\zeta')\Big)\Big]\pi^\zeta(\zeta'|\zeta)(\mathrm{d}\zeta')\Big\}.$$

where $\boldsymbol{g}' \equiv \{\zeta' \in P : g' \in K\}$, $\boldsymbol{\delta} \equiv \{\zeta' \in P : \delta_{g'(\zeta')}\}$ and $\boldsymbol{\varrho} \equiv \{\zeta' \in P : \varrho_{\delta_{g'(\zeta')}}(g'(\zeta'))\}$,

The proposition is made of two parts. First, if the second derivative of the cost of distribution is constant, then $\Pi_{\pi^g(\zeta')}$ is strictly quasiconcave. The reason is that both $V_0^b(x,s)$ and $V^l(x,s)$ are concave in s. Second, when $\Pi_{\pi^g(\zeta')}$ is strictly quasiconcave, the problem of choosing an optimal distribution is the same as the one of choosing g' directly. In other words, the optimal distribution choice collapses to a Dirac distribution of ζ-contingent sequence of g'.

4.3 Moral Hazard and Limited Enforcement

In the economies we study, with the need of risk-sharing, avoiding default or undesired permanent transfers, moral hazard problems arise when these problems could be alleviated with appropriate shock distribution, but such distribution is not contractible. The following lemma provides a characterization of the interaction between limited enforcement and moral hazard constraints.

Lemma 2 *When $g' = \underline{g}$, then $\varrho_{\pi^g(\zeta')}(s') = 0$. Otherwise, $\varrho_{\pi^g(\zeta')}(s') > 0$.*

The lemma states that when the highest g' realizes, the multiplier attached to the ICC is zero. Otherwise, it is strictly positive. The rationale behind this result is the following. The borrower incurs zero cost when $\underline{g}$ realizes. However, for any $g' > \underline{g}$, the cost is positive and the borrower needs to be compensated accordingly. Hence, when $g' = \underline{g}$, the borrower only gets the basis $V_0^b(x',\zeta')$ meaning that $\varrho_{\pi^g(\zeta')}(s') = 0$. For any other realization, $\varrho_{\pi^g(\zeta')}(s') > 0$ to compensate the borrower for incurring more costs.

A direct corollary is that the law of motion of the relative Pareto weight is a left bounded positive submartingale. To see this, take the expectations of the law of motion

$$\mathbb{E}_t x_{t+1}(s^{t+1}) \equiv \mathbb{E}_t\left[\bar{x}_{t+1}(s^t) + \hat{x}_{t+1}(s^{t+1})\right] = \mathbb{E}_t\left[\frac{1+\nu_{b,t}(s^t)}{1+\nu_{l,t}(s^t)}x(s^t) + \frac{\varrho_{\pi^g_{t+1}}(s^{t+1}|s^t)}{1+\nu_{l,t}(s^t)}x(s^t)\right]\eta,$$

where $\hat{x}_{t+1}(s^{t+1})$ accounts for the dynamic effect of the moral hazard constraint. There are two forces working against the effect of the ICC multiplier: impatience $\eta < 1$ and the lender's LE constraint $\nu_{l,t}(s^t) \geq 0$. Hence, with neither the lender's LE constraint and nor impatience, we get that $\mathbb{E}_t x_{t+1}(s^{t+1}) \geq x_t(s^t)$. This means that without either one of these two elements, the relative Pareto weight would go towards infinity. This is the reverse of the *immiseration* result of Atkeson and Lucas (1992). We call it *bliss*.

Corollary 1 (Bliss) *When $\eta = 1$ and $\nu_{l,t}(s^t) = 0$ in all states, $\mathbb{E}_t x_{t+1}(s^{t+1}) \geq x_t(s^t)$.*

This result is important as it uncovers a different provision of incentives as the canonical moral hazard problem. In the next section we analyze such problem and contrast it with the outcome of Lemma 2.

5 The Fund under Canonical Moral Hazard

We now switch to the canonical MH. The borrower provides effort at a cost in order to improve – in the first-order stochastic sense – the distribution of g. Effort is not contractible and affects the distribution globally. As a result, the borrower cannot anymore manipulate in an arbitrary way the relative likelihood of any collection of g shocks.

5.1 The Constraints

The borrower's choice of distribution is restricted to a family of distributions in $\mathcal{M}$. In particular, define Q as a mixture distribution $Q = \varpi(e)Q_L + (1 - \varpi(e))Q_H$ for $Q_L, Q_H \in \mathcal{M}$ with weighting function $\varpi(e)$. The borrower can manipulate the weights by choosing the (non-contractible) effort $e \in [0, 1]$ to change Q. We assume that Q_H first-order stochastically dominates Q_L, $\varpi'(e) < 0$ and $\varpi''(e) < 0$.

To recover the formulation of the canonical moral hazard problem, we consider a different effort cost than what we had so far. The cost of effort $\hat{v} : [0, 1] \to \mathbb{R}$ is a mapping from $[0, 1]$ instead of $\mathcal{M}$. We then have that $\hat{U}(c, n, e) = u(c) + h(1 - n) - \hat{v}(e)$. Additionally, we assume independence between ζ and g. This implies that a single shock variable, g, depends on effort. We denote the joint distribution of ζ and g by $\Upsilon(s'|s, e)$.

Notice that the shape of the cost function $\hat{v}(e)$ prevents us to use the apparatus developed previously. In the next chapter we analyze the case of restricted distribution with the previously adopted cost function $v(\pi^g)$.

Given the re-definition of the utility function – which is now defined over e instead of π^g – the

LE constraint of the lender is given by (3), while the LE constraint of the borrower reads

$$\mathbb{E}_t\left[\sum_{j=t}^{\infty}\beta^{j-t}U(c(s^j),n(s^j),e(s^j))\right]\geq V^D(s_t). \tag{14}$$

The main change relates to the *incentive compatibility constraint* (ICC). Instead of choosing an entire distribution, the borrower picks an effort level. Define $\hat{V}^b(s^t)=\mathbb{E}_t[\sum_{j=0}^{\infty}\beta^j\hat{U}(c(s^{t+j}),n(s^{t+j}),e(s^{t+j}))]$ as the value of the borrower at time t. The optimal choice of effort is given by

$$e(s^t)=\underset{\tilde{e}}{\operatorname{argmax}}\ \hat{U}(c(s^t),n(s^t),\tilde{e})+\beta\int\int\hat{V}^b(s^{t+1})\Upsilon(s^{t+1}|s_t,\tilde{e})(\mathrm{d}g^{t+1})(\mathrm{d}\zeta^{t+1}).$$

The ICC is therefore

$$\hat{v}_e(e(s^t))=\beta\int\int\hat{V}^b(s^{t+1})\frac{\partial\Upsilon(s^{t+1}|s_t,e(s^t))}{\partial e(s^t)}(\mathrm{d}g^{t+1})(\mathrm{d}\zeta^{t+1}). \tag{15}$$

The difference with respect to the case of flexible moral hazard is that g is assumed to follow a fixed Markovian distribution given e.

For the first-order approach of Rogerson (1985) to be valid, the cumulative distribution function of g should be differentiable, convex and satisfy the monotone likelihood-ratio condition. This mirrors our Assumption 1. Moreover, the ICC in (15) relies on the informativeness of the realization of g. This is because the information content of a specific realization can be directly measured by the relative likelihood given that we know Υ. In the case of flexible moral hazard, the probability distribution is a choice variable which is not contractible. This precludes any informativeness of the shock realization.

Assumption 4 (*Differentiability, Monotonicity and Convexity*) *For every s, if $e\geq\tilde{e}>0$ the ratio $\frac{\Upsilon(s'|s,\tilde{e})}{\Upsilon(s'|s,e)}$ is nonincreasing in g', and, for every (e,s), $F_p(e,s)=\int_p\Upsilon(\{\zeta',g'\}|s,e)(\mathrm{d}\zeta')$ with $p\subseteq P$ is differentiable in e, with $\partial_e F_p(e,s)\leq 0$ and $\partial_e^2F_p(e,s)\geq 0$,.*

Assumption 4 generalizes the assumptions of Rogerson (1985) so that we can apply his first-order condition approach in a simple static Pareto-optimization problem to our dynamic contracting problem with limited enforcement and moral hazard frictions.

5.2 The Long Term Contract

With *two-sided limited enforcement* and *moral hazard*, an optimal Fund contract is a solution to the following **Fund problem**:

$$\max_{\{c(s^t),n(s^t),e(s^t)\}}\mathbb{E}_0\left[\alpha_{b,0}\sum_{t=0}^{\infty}\beta^t\hat{U}(c(s^t),n(s^t),e(s^t))+\alpha_{l,0}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t\left[\theta_t f(n(s^t))-c(s^t)\right]\right]$$
$$\text{s.t.}\quad (3),\ (14),\ \text{and } (15),\ \forall s^t,t\geq 0.$$

In terms of structure, the Fund problem is very similar to what we had under flexible moral hazard. The main change is that the government exercises effort $e(s^t)$ instead of directly choosing a distribution π^g_{t+1}. As before, to ensure the uniform boundedness of the Lagrange multipliers, we posit an interiority assumption.

Assumption 5 (*Interiority*) *There is an $\epsilon > 0$, such that, for all $s_0 \in S$ there is a program $\{\tilde{c}(s^t), \tilde{n}(s^t), \tilde{e}(s^t)\}_{t=0}^{\infty}$ satisfying constraints (14) and (3) when, on the right-hand side, $V^D(s_t)$ and $Z(s_t)$ are replaced by $V^D(s_t) + \epsilon$ and $Z(s_t) + \epsilon$, respectively, and similarly, when in (15) $\partial_e \hat{v}(e(s^t))$ is replaced by $\partial_e \hat{v}(e(s^t)) + \epsilon$ and $=$ is replaced by $\leq$.*

The interiority of effort can be guaranteed if full risk sharing is not the only feasible allocation and appropriate Inada conditions are imposed on the cost $\hat{v}(e)$ and benefit $\Upsilon(s'|s,e)$ of effort. The following assumption parallels Assumption 2 for the economy with canonical moral hazard.

Assumption 6 (Monotonicity of $V^D(s)$ with respect to the s partial-order) *If $\zeta \geq \zeta'$ and $g \leq g'$ then $V^D(s) \geq V^D(s')$, with inequality if, in addition, $s \neq s'$.*

Following the previous section, we can formulate the Fund problem in recursive form. We find that the *Saddle-Point Functional Equation* (**SPFE**) — i.e. the saddle-point version of Bellman's equation — is given by

$$\hat{FV}(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ x\Big[(1+\nu_b)\hat{U}(c,n,e) - \nu_b V^D(s) - \varrho \hat{v}_e(e)\Big] \tag{16}$$

$$+ \big[(1+\nu_l)(\theta(s)f(n) - c) - \nu_l Z\big] + \frac{1+\nu_l}{1+r} \int\int \hat{FV}(x',s')\Upsilon(s'|s,e)(\mathrm{d}g')(\mathrm{d}\zeta') \Big\}$$

$$\text{s.t. } x'(s') \equiv \bar{x}'(s) + \hat{x}'(s') = \left[\frac{1+\nu^b}{1+\nu_l} + \frac{\varphi(s'|s)}{1+\nu_l}\right]\eta x \text{ and } \varphi(s'|s) = \varrho \frac{\partial_e \Upsilon(s'|s,e)}{\Upsilon(s'|s,e)}. \tag{17}$$

The Fund's value functions can be decomposed as in the case with flexible moral hazard. Similarly, the policy functions for consumption and labor of the Fund contract are the solution to (11) and (12). This is because of additive separability in the utility function. The formulation of the moral hazard does not directly affect the optimal choice of consumption and labor.

Notice that the multiplier $\varphi(s'|s)$ is defined as $\varrho \frac{\partial_e \Upsilon(s'|s,e)}{\Upsilon(s'|s,e)}$. It does not explicitly depend on e since, as multiplier, the action is taken as given. Moreover, it can be positive or negative depending on the sign of $\partial_e \Upsilon(s'|s,e)$. This reflects the main difference with the flexible moral hazard approach we discussed previously. As the choice of distribution is restricted, the relative likelihood is informative about the realization of g'.

The effort policy $e(x,s)$ is determined by the first order condition of the SPFE with respect to

e, which can be conveniently expressed as:

$$\begin{aligned}\hat{v}'(e(x,s)) = {} & \beta \int\int \partial_e \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ & + \frac{1+\nu_l(x,s)}{1+\nu_b(x,s)}\frac{1}{x}\frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ & - \frac{\varrho(x,s)}{1+\nu_b(x,s)}\left[v''(e(x,s)) + \beta\int\int \partial_e^2 \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta')\right]. \end{aligned} \tag{18}$$

Equation (18) balances the marginal cost of effort with the benefits. The first line is the life-time utility benefit of effort to the borrower; the second line is the marginal benefit of effort to the lender, in terms of the borrower's marginal utility, given by (11); the third line accounts for the marginal relaxation/tightening effect of the moral hazard constraint (4) when there is a change in effort. With contractible effort, the Fund problem would not have the *incentive compatibility constraint* (4) and the effort decision would be given by the first two lines, with the second one accounting for the social value of effort. In contrast, with non-contractible effort, as we assume, constraint (4) is present and the first line is equal to zero, namely:

$$\hat{v}'(e(x,s)) = \beta \int\int \partial_e \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta'). \tag{19}$$

In this case, (18) reduces to

$$\begin{aligned} & \frac{1}{1+r}\int\int \partial_e \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta') \\ & \qquad = \vartheta(x,s)\left[v''(e(x,s)) - \beta\int\int \partial_e^2 \Upsilon(s'|s,e)\hat{V}^b(x'(s'),s')(\mathrm{d}g')(\mathrm{d}\zeta')\right], \end{aligned} \tag{20}$$

where $\vartheta(x,s) \equiv \frac{x\varrho(x,s)}{1+\nu_l(x,s)}$ can be interpreted as the marginal value of relaxing the ICE constraint in terms of the lender's valuation; that is, (20) accounts for the external effect of effort on the lender's value through its effect on the incentive compatibility constraint. Note that, although incentive compatibility implies that only the borrower's returns affect the effort decision directly, the benefits represented in (20) will affect incentives as they affect $\varrho(x,s)$ and hence the whole future path of allocations through (17).

5.3 Limited Enforcement and Moral Hazard

In the economies we study, with the need of risk-sharing, avoiding default or non sustainable debt levels, moral hazard problems arise when these problems could be alleviated with effort, but such effort is not contractible. Therefore it is reasonable to model limited enforcement contracts that satisfy the following property:

Definition 2 *The limited enforcement constraints* (14) *and* (3) *satisfy the 'no-free-lunch condition' if, given* (x, s)*, whenever* $\nu_b(x'(x, s), s') > 0$*, then* $\partial_e \Upsilon(s' \mid s, e) > 0$ *and whenever* $\nu_l(x'(x, s), s') > 0$*, then* $\partial_e \Upsilon(s' \mid s, e) < 0$*, respectively.*

Conversely, if $\partial_e \Upsilon(s' \mid s, e) = 0$ (or the inequality signs were reversed) exercising more effort would not have any effect on the limited enforcement constraints (or a perverse effect) and, on those grounds, moral hazard would not be an issue. The following lemma provides a characterization of the interaction between limited enforcement and moral hazard constraints. Before we state it, note that the law of motion of x which can be written as:

$$x_{t+1}(s^{t+1}) \equiv \left[\bar{x}_{t+1}(s^t) + \hat{x}_{t+1}(s^{t+1})\right] = \left[\frac{1 + \nu_{b,t}(s^t)}{1 + \nu_{l,t}(s^t)} x(s^t) + \frac{\varphi_{t+1}(s^{t+1}|s^t)}{1 + \nu_{l,t}(s^t)} x(s^t)\right] \eta,$$

where $\hat{x}_{t+1}(s^{t+1})$ accounts for the dynamic effect of the moral hazard constraint.

Lemma 3

i) In recursive contracts, limited enforcement constraints have an effect on the expected law of motion of the Pareto weights, when they are binding; in contrast, moral hazard constraints do not have an effect on $\{\mathbb{E}_t x_{t+1}\}$*, even if they bind; i.e.* $\mathbb{E}_t x_{t+1} = \bar{x}_{t+1}(s^t)$*.*

ii) If limited enforcement constraints satisfy the 'no-free-lunch condition', moral hazard constraint make no-default constraints (14) *more likely to bind and sustainability constraints (3) less likely to bind and, in both cases,* $\mathbb{E}_t \frac{1}{u'(c_{t+1})}$ *increases.*

To see *i)*, note that, given the above decompostion of the law-of-motion of x, $\sum_{s^{t+1}|s^t} \Upsilon(s^{t+1}|s^t, e(s^t))\hat{x}_{t+1}(s^{t+1}) = 0$, since independently of effort $\sum_{s^{t+1}|s^t} \Upsilon(s^{t+1}|s^t, e(s^t)) = 1$, hence $\sum_{s^{t+1}|s^t} \partial_e \Upsilon(s^{t+1}|s^t, e(s^t)) = 0$. Therefore $\mathbb{E}_t \hat{x}_{t+1} = 0$ and $\mathbb{E}_t x_{t+1} = \bar{x}_{t+1}(s^t)$. Alternatively, the expected law of motion of x can also be expressed as

$$\mathbb{E}_t x_{t+1} = \mathbb{E}_t \left[\frac{1}{u'(c_{t+1})} \frac{1 + \nu_{l,t+1}(s^t)}{1 + \nu_{b,t+1}(s^t)}\right] = \frac{1}{u'(c_t)} \eta,$$

where the last equality is the *inverse Euler equation* of the recursive contract (Ábrahám et al. (2025), Lemma 4).

To see *ii)*, note that, since the limited enforcement multipliers are either zero or at most one of the two is positive, we can have the following decomposition

$$\mathbb{E}_t \frac{1}{u'(c_{t+1})} = \mathbb{E}_t \left[x_{t+1} \frac{1 + \nu_{b,t+1}(s^{t+1})}{1 + \nu_{l,t+1}(s^{t+1})}\right] = \mathbb{E}_t x_{t+1} + \mathbb{E}_t x_{t+1} \nu_{b,t+1}(s^t) - \mathbb{E}_t x_{t+1} \frac{\nu_{l,t+1}(s^t)}{1 + \nu_{l,t+1}(s^t)},$$

where $\mathbb{E}_t x_{t+1} = \eta x_t$ and, without incentive constraints, the last two terms simply denote the change in the relative Pareto weight when either the no-default or the sustainability constraints binds. However, if limited enforcement constraints satisfy the 'no-free-lunch condition', the no-default constraint is more likely to bind, while the sustainabiliy constraint is less likely to bind and, as a result, in both cases expected consumption increases.

6 Flexible vs. Canonical Moral Hazard

In this section, we contrast the two moral hazard formulations. In particular, we show that the expression for the cost of effort is key for the provision of incentives. Even if the choice of distributions is restricted, the Fund may function as in the case of flexible moral hazard. For this, the cost of effort has to incorporate the properties of the probability distribution.

6.1 Back-loaded moral hazard

In the canonical moral hazard problem, the provision of incentives is generated by a system of rewards and punishment associated with the moral-hazard constraint (15). Such system is not unique. We analyze contracts that consist of an infinite sequence of subprograms, whereby within each subprogram rewards and punishment are *back-loaded* to the end.

We consider a contract with subprograms. The length of such subprograms is directly determined by the binding limited enforcement constraints. The reason is that whenever a subprogram violates one of the limited enforcement constraints, one of the contracting parties would find it optimal to terminate the contract. Hence, the binding limited enforcement constraints endogenously determine the subprogram's length.

The Fund contract can be expressed as the solution to a sequence of recursive (SPFE) problems. As the program resets when one of the limited enforcement constraint binds, the start of a subprogram is such that

$$\begin{aligned}
\hat{FV}(x,s) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ & x\big[(1+\nu_b)(u(c)+h(1-n)-\hat{v}(e)) - \nu_b V^0(s) - \varrho v'(e)\big] \\
& + [(1+\nu_l)(\theta f(n)-c) - \nu_l Z] \\
& + \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s')\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s')\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x'(s') = \eta x \frac{1+\nu_b+\psi(s' \mid s,e)}{1+\nu_l} \\
& \bar{x}' = \eta x \frac{1+\nu_b}{1+\nu_l}
\end{aligned}$$

where $\mathbb{I}_{\{(x',s')\}}$ is an indicator function where $\mathbb{I}_{\{(x',s')\}} = 1$ if one of the limited enforcement constraints is binding – i.e. $\nu_b(x',s') + \nu_l(x',s') > 0$. Then within the subprogram

$$\begin{aligned}
\overline{FV}(x,s;\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ & \bar{x}\,[u(c)+h(1-n)] - x\big[\hat{v}(e)+\varrho v'(e)\big] + (\theta f(n)-c) \\
& + \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s')\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s')\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x' = \eta x\left[1+\psi(s' \mid s,e)\right] \\
& \bar{x}' = \eta\bar{x},
\end{aligned}$$

As it can be seen, within the subprogram x' is the *latent aggregated multiplier* and the consumption and labor policies are given by the same first-order condition, (11) and (12), resulting in $c(\overline{x}, s)$ and $n(\overline{x}, s)$, while the optimal effort $e(\overline{x}, s)$ requires a reformulation of (20). For this, it is useful to recall that, as in the benchmark Fund contracts, $\hat{FV}(x, s) = x\hat{V}^b(x, s) + V^l(x, s)$. However, $\overline{FV}(x, \overline{x}, s)$ depends on the within the subprogram relative Pareto weight $\overline{x}$ and the *latent relative Pareto weight* x; therefore, we first decompose

$$\begin{aligned}
\overline{V}^b_1(\overline{x}, s) &= u(c(\overline{x}, s)) + h(1 - n(\overline{x}, s)) + \beta\mathbb{E}\overline{V}^b_1(x', s'), \\
\overline{V}^b_2(x, s) &= -\hat{v}(e(x, s)) + \beta\mathbb{E}\overline{V}^b_2(x', s'),
\end{aligned}$$

then the value of the borrower is simply $\overline{V}^b(x, s, \overline{x}) = \overline{V}^b_1(\overline{x}, s) + \overline{V}^b_2(x, s)$ implying that

$$\overline{FV}^b(x, s, \overline{x}) = \overline{x}\overline{V}^b_1(\overline{x}, s) + x\overline{V}^b_2(x, s) + \overline{V}^l(x, s, \overline{x}).$$

Note that, except for the distinction of $\overline{x}$, $\overline{FV}(x, s; \overline{x})$ is the same as $\hat{FV}(x, s)$ since we can always incorporate in the minimization $\{\nu_b, \nu_l\}$ which will satisfy $\nu_b = \nu_l = 0$, by construction, within the subprogram.

In Appendix A, we provide some more details about the behavior of consumption and labor in a subprogram. We further consider additional stochastic and deterministic criteria determining the subprogram length.

6.2 Restricted flexible moral hazard

Besides the back-loading of incentives, there is another way to bridge the gap between the flexible and the canonical MH. In Section 4, the cost of effort is given by $v(\pi^g)$ which is a mapping from $\mathcal{M}$ to the real. In Section 5, the cost of effort becomes $\hat{v}(e)$ which is a mapping from $[0, 1]$ to the real. This cost function does not incorporate any information about the distribution implied by the choice e.

Now if we were to re-formulate the cost function in the canonical moral hazard case to get closer to the flexible moral hazard, we would get

$$v(Q_e) = K\left[\int (g - \underline{g})Q_e(\mathrm{d}g)\right],$$

where $Q_{\tilde{e}} = \varpi(\tilde{e})Q_L + (1 - \varpi(\tilde{e}))Q_H$ is the distribution obtained for a specific effort $e = \tilde{e}$. This cost function integrates the properties of the chosen distribution consistent with Assumption 1. Note as well that as Q is a mixture distribution, the family of distribution is convex.

Clearly, one can formulate an IC constraint as we did in Section 4. This follows from the Gateau differentiability of the cost function v. In other words, full flexibility in the choice of the distribution is not required to formulate the IC constraint as we did. The Fund contract in recursive form reads as follows

$$FV(x,s) = \text{SP} \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Bigg\{ x\Big[(1+\nu_b)U(c,n,Q(e)) - \nu_b V^D(s)\Big] \tag{21}$$

$$+ \Big[(1+\nu_l)[\theta f(n) - c] - \nu_l Z(s)\Big]$$

$$+ \int\int \left[\frac{1+\nu_l}{1+r} FV(x'(s'),s') - x\varrho\Big(v_{Q(e)}(g') + V_0^b(x',\zeta')\Big)\right] \Upsilon(s'|s,e)(\mathrm{d}g')(\mathrm{d}\zeta') \Bigg\}$$

$$\text{s.t.} \quad x'(s') = \left[\frac{1+\nu_b}{1+\nu_l} + \frac{\varrho'}{1+\nu_l}\right]\eta x. \tag{22}$$

Two main elements deserve to be noted. First, the choice of effort pins down a non-contingent distribution. This differs from the Fund in Section 4 in which the borrower could choose a distribution conditional on ζ'. Second, the provision of incentive does not rely on realized outcomes. In particular, there is no punishment in the case of a bad outcome. As one can see in (22), MH can only increase the relative Pareto weight. This means the main takeaway of Section 4 holds true.

7 Quantitative Analysis

In this section, we present the parametrization of the model and the results of the quantitative analysis. In future versions, we will properly calibrate the model to match specific moments.

7.1 Parametrization

We parametrize the model following Ábrahám et al. (2025). The utility of the borrower is additively separable in consumption, leisure and effort. In particular, we assume that $u(c) = \log(c)$ and $h(1-n) = \gamma\frac{(1-n)^{1-\sigma}-1}{1-\sigma}$. For the flexible moral hazard, we consider $v(\pi^g) = \omega[\int(g-\underline{g})\pi^g(\mathrm{d}g)]^2$ and for the canonical moral hazard $v(e) = \omega e^2$ so that the third derivative is zero in both cases.

The preference parameters are set to $\sigma = 0.6887$, $\gamma = 1.4$, $\omega = 0.1$ and the discount factor to $\beta = 0.945$. The risk free interest rate is set to $r = 2.48\%$, the average short-term real interest rate of Germany. The fact that the borrower is less patient than the lender implies that the borrower would like to front-load consumption.

Table 1: Parameter Values

α	β	σ	γ	r	λ	ψ	δ	ω	Z
0.566	0.945	0.6887	1.4	0.0248	0.1	0.8099	0	0.1	0

The LE constraint of the lender is set to $Z = 0$, implying no expected permanent transfers between the borrower and the lender at any time or state. In other words, the Fund is not built on an assumption of solidarity which would require permanent transfers.

We assume that *Fund-exit is irreversible*, with the interpretation that the fund can commit to exclusion of the borrower. If a country defaults, it is also subject to an asymmetric default penalty as in Arellano (2008)

$$\zeta^p = \begin{cases} \bar{\zeta}, & \text{if } \zeta \geq \bar{\zeta} \\ \zeta, & \text{if } \zeta < \bar{\zeta} \end{cases} \quad \text{with } \bar{\zeta} = \psi \mathbb{E}\zeta,$$

where $\psi = 0.8099$. In addition, we adopt a probability to re-enter the market of 0.1. To simplify computation, we set $\delta = 0$ in the incomplete market economy with default.

Regarding the technology, we assume that $f(n) = n^\alpha$ with the labor share of the borrower set to $\alpha = 0.566$ to match the average labor share across the Euro Area 'stressed' countries. Table 1 summarizes the parameter values.

Regarding the calibration of the productivity shocks, we consider that $\zeta \in \{0.81, 1.01, 1.12\}$ and the transition matrix is given by

$$\pi = \begin{bmatrix} 0.980 & 0.015 & 0.005 \\ 0.005 & 0.975 & 0.020 \\ 0.015 & 0.025 & 0.960 \end{bmatrix} \tag{23}$$

Regarding the shock g, we allow for 3 realizations $g \in \{0.0785, 0.0415, 0.0185\}$. For the flexible moral hazard, this is all we need to define as the borrower is free to pick any distribution over the vector of g. However, for the canonical moral hazard, the borrower is restricted in the choice of distribution. Hence, we additionally need to determine Q.

Recall that $Q = \varpi(e)Q_L + (1 - \varpi(e))Q_H$ for $Q_L, Q_H \in \mathcal{M}$. Effort affects the probability distribution over next period's realisation of g'. We consider that Q_L first-order stochastically dominates Q_H and $\varpi'(e) < 0$ and and $\varpi''(e) < 0$. For this, we set $\varpi(e) = (e - 1)^2$ and

$$Q_L = \begin{bmatrix} 0.93 & 0.0466 & 0.0234 \\ 0 & 0.99 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}, \ Q_H = \begin{bmatrix} 1 & 0 & 0 \\ 0.06 & 0.94 & 0 \\ 0.03 & 0.04 & 0.93 \end{bmatrix}$$

This functional form implies simple expressions for $\frac{\partial Q(g'|g,e)}{\partial e}$ and $\frac{\partial^2 Q(g'|g,e)}{\partial e^2}$ as follows:

$$\frac{\partial Q(g'|g,e)}{\partial e} = -\varpi'(e)\big[Q_L(g'|g) - Q_H(g'|g)\big] = 2(1-e)\big[Q_L(g'|g) - Q_H(g'|g)\big]$$
$$\frac{\partial^2 Q(g'|g,e)}{\partial e^2} = -\varpi''(e)\big[Q_L(g'|g) - Q_H(g'|g)\big] = -2\big[Q_L(g'|g) - Q_H(g'|g)\big].$$

Note that with this functional forms Assumption 4 is satisfied.

7.2 Policy functions

In this subsection, we expose the main policies associated to the Fund problem. Especially, we present the policies for the relative Pareto weight, consumption, labor and g. We first focus on the flexible moral hazard before going to the canonical moral hazard.

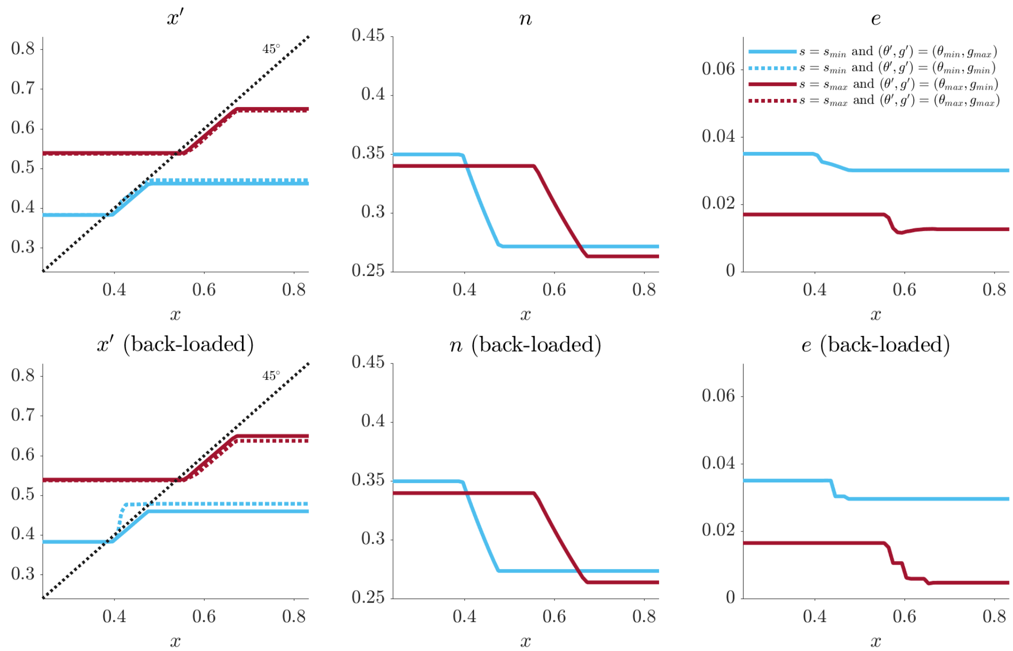


Figure 1: Main Policy Functions – Canonical and Back-loaded MH

Figure 1 depicts the main policies related to the canonical moral hazard as a function of the relative Pareto weight x. In particular, we distinguish between the standard provision of incentives and the back-loaded one. The red lines (either solid or dotted) relate to the highest value of s and the blue lines to the lowest value of s. For each color, the solid line corresponds to a transition to the same g, while the dotted corresponds to a transition to the opposite one.

In each specification, the horizontal line on the left hand side is determined by the borrower's binding LE constraint, while the horizontal line on the right hand side is determined by the Fund's binding LE constraint. The line rejoining both horizontal lines is determined by the first best allocation and has a slope of $\eta < 1$.

As we assume log-utility, $c = x'/\eta$. We therefore do not depict the policy for consumption as it closely follows the relative Pareto weight. In opposition, labor follows the inverse direction. As the relative Pareto weight increases, consumption increases but labor reduces if the constraints are not binding, however as it can be seen in Figure 1 when the borrowing constraint is binding at low values of x and ζ, there may be a reversal: n is higher in our simulation, signaling that even with the Fund, there may be some austerity at these low values. These results follow from the first-order conditions in (11) and in (12).[6]

We find significant differences in the law of motion of the relative Pareto weights between the

[6] See Lemma 1 in Ábrahám et al. (2025); in contrast with them, our simulations have the indicated reversal effect on n with respect to ζ when the borrower's constraint binds at low values of s, while we do not have -as they do – a reversal effect of e with respect to g when the lender's constraint binds at high values of s.

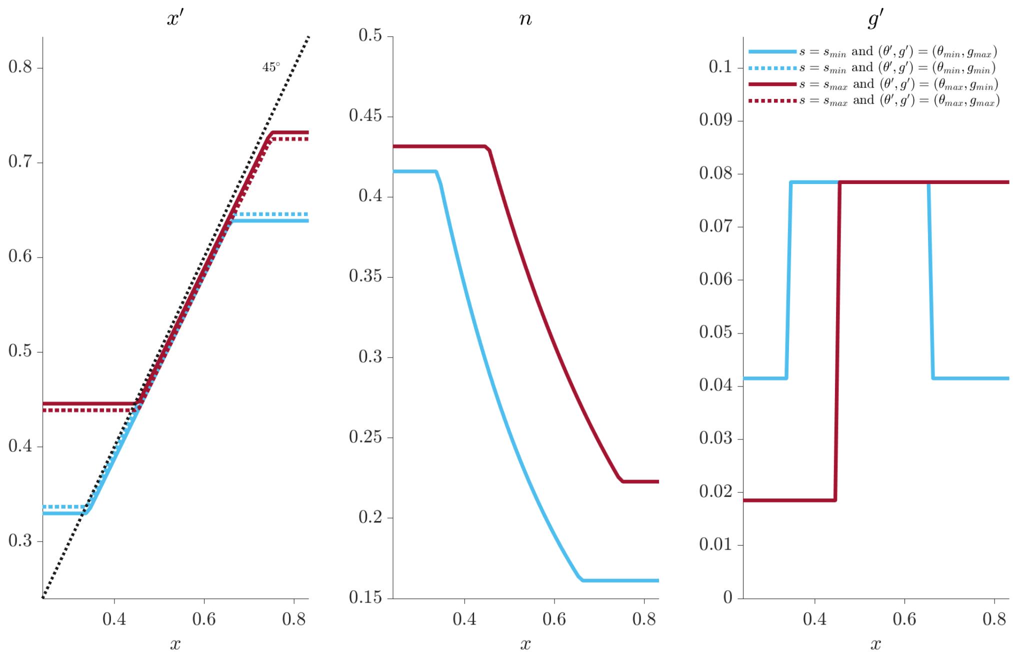


Figure 2: Main Policy Functions – Flexible MH

two specifications. In particular, in the back-loaded case, we note a larger wedge between the solid and the dotted line of each color. This means that the relative Pareto weight reacts more strongly to changes in g. Regarding consumption and labor, we observe little changes. This is because the determination of such variables does not directly depend on the value of the multiplier attached to the ICC, ϱ.

Figure 2 depicts the main policies related to the flexible moral hazard as a function of the relative Pareto weight x. The figure uses the same color system as the previous one.

In terms of relative Pareto weight, we observe two elements. First, when the borrower picks a $g' > g$, then the relative Pareto weight decreases. In opposition, when the borrower picks $g' < g$, the opposite holds true. This is because the Fund has to compensates the borrower when it chooses a lower g' because it is more costly to implement.

As consumption follows the same dynamic as the relative Pareto weight, we do not depict it. Moreover, as in the canonical moral hazard, labor reduces as the relative Pareto weight increases. This is because the Fund with flexible and canonical moral hazard, lead to the same first-order conditions with respect to c and n.

Finally, looking at the policy function for g', we observe that the level of shocks is not necessarily monotonic in x. This is due to the lender's LE constraint as shown in Ábrahám et al. (2025). This non-monotonicity disappears once we reomve the lender's LE constraint.

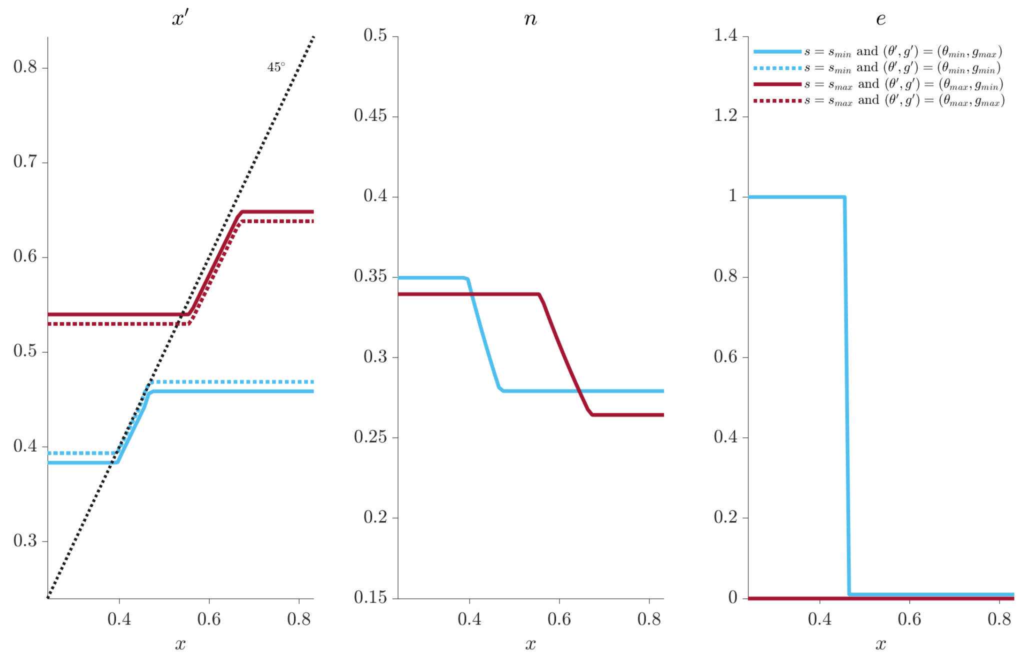


Figure 3: Main Policy Functions – Restricted Flexible MH

Figure 3 depicts the main policies related to the restricted flexible moral hazard as a function of the relative Pareto weight x. The main difference with the flexible moral hazard is that the borrower is restricted to choose the distribution of g' among a specific family of distribution Q.

We observe a similar dynamic of the relative Pareto weight as in Figure 2. Moreover, we directly see that the borrower provides more effort than what is depicted in Figure 1. This is because the cost structure internalizes the cost of choosing a specific distribution.

7.3 Steady state analysis

We analyze the Fund's allocation in steady state. We compute welfare gains with respect to the Fund with canonical MH and distinguish two cases. First, the Fund contracts which have outside options given by (1). Second, we re-compute the different Fund contracts with the same outside option.

Figure 4 depicts the ergodic set of the relative Pareto weights for the different Fund contracts. The definition of the ergodic set is given in Definition A.1. The dark grey region corresponds to the ergodic set, while the light grey region is the basin of attraction. The dark grey region is therefore the steady state in which we will contact simulation exercises.

We observe only little differences in the determination of the ergodic set of the different specifications. The ergodic set of the Fund under flexible MH has a slightly narrower width, though.

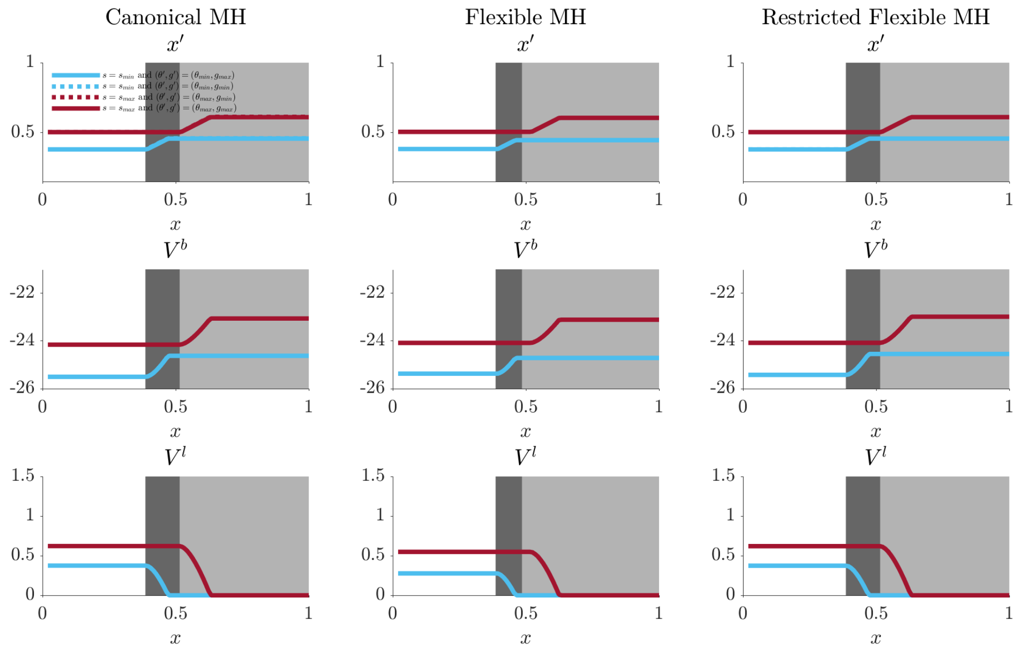


Figure 4: Ergodic Set – Canonical and Back-loaded MH

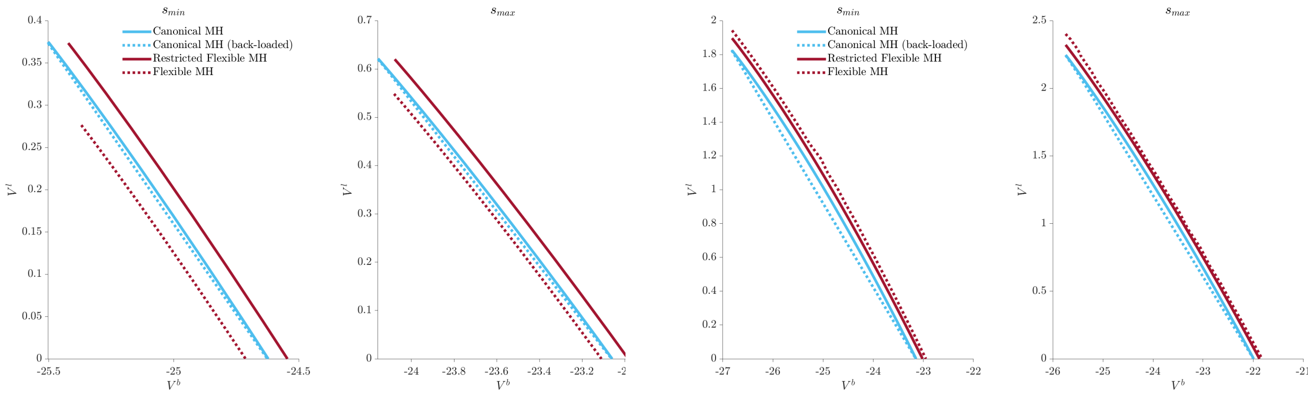
Figure 5a depicts the Pareto frontiers for the different Fund contracts with outside options given by (1) (i.e. IMD outside option). We see that the Fund contract under flexible MH is Pareto dominated by all the other contracts. The most efficient contract is the restricted flexible MH.

Figure 5b depicts the Pareto frontiers for the different Fund contracts with the same outside options. The IMD outside option given by (1) differ with the MH framework. That is why we re-compute all the contracts with the same outside option. We see that the Fund contract under flexible MH Pareto dominates all the other contracts when the outside option is the same. This is the opposite of what Figure 5a shows. It comes from the fact that the flexible MH leads to a higher value of the outside option, which is more costly in terms of resource the Fund must provide.

We complement this with a welfare analysis in steady state presented in Table 2. Welfare is calculated as a percent of consumption-equivalent changes. Results are consistent with Figures 5a and 5b. Under the IMD outside option, the welfare losses under flexible MH come from the Fund side. The borrower is actually better off given the larger outside option.

8 Conclusion

From the perspective of economic theory, since the pioneer work of Prescott and Townsend (1984) it is understood that under appropriate convexity assumptions moral hazard (and adverse selection) problems can be incorporated in the problem of efficiently assigning resources subject to technologi-



(a) IMD outside option (b) Same outside option

Figure 5: Pareto Frontiers

cal and feasibility constraints, by introducing Incentive Compatibility (IC) constraints in parallel to other constraints. Furthermore, it is also understood that under these, and other standard assumptions, the corresponding competitive equilibrium exists and the First and Second Welfare Theorems are satisfied for constrained-efficient allocations. Extensive follow up work has extended their results to dynamic economies – e.g. with debt or other financial assets, etc. However, not much work has been done in studying different forms of implementation; for example, in dynamic economies with debt and risk-sharing contracts. In particular, given its novel nature, to our knowledge, no work has been done extending the *flexible moral hazard* approach to dynamic contracts – hence, to recursive contracts with limited enforcement constraints. Here, we have pursued this enquire and have shown how different forms to implement IC constraints result in different constraint-efficient allocations and, correspondingly, in a different split of the surplus between the risk-averse borrower and the risk-neutral lender, while satisfying limited enforcement (LE) constraints. The *restricted* and *back-loaded* design help to narrow the gap between the theory and the current practices of official lenders who have a long-term relation with their borrowing countries (as it is the case with the ESM and euro area countries), while our development of the *flexible moral hazard* approach opens a new venue for the design of official lending programs, since in assessing the risk-profile of a country the first question that arises is: what is its capacity, and cost, to choose a better risk distribution? In fact, conditional reforms to prevent pandemics or natural disasters are choices of distributions.

Table 2: Welfare Gains

Outside option: IMD					
Borrower			Fund		
Canonical (back-loaded)	Flexible	Restricted flexible	Canonical (back-loaded)	Flexible	Restricted flexible
0.04	0.62	0.56	-0.01	-1.69	-0.09
Outside option: same for all					
Borrower			Fund		
Canonical (back-loaded)	Flexible	Restricted flexible	Canonical (back-loaded)	Flexible	Restricted flexible
0.03	1.21	0.23	0.00	2.67	1.43

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Appendix

A Back-Loaded Moral Hazard

We first now give the characteristics of a subprogram in terms of length, consumption and labor.

The long-run property of the Fund contract is related to the definition of an ergodic set of relative Pareto weights, x. The term ergodic refers to the fact that the contract will move around the same set of relative Pareto weights over time and over histories.

Definition A.1 (Steady State) *A* Steady State Equilibrium *is defined by an ergodic set* $X^{ss} \equiv [x^{lb}, x^{ub}]$ *where the lower bound is* $x^{lb} = \min_{s\in S}\{x : V^b(x,s) = V^D(s)\}$ *and the upper bound is* $x^{ub} = \max_{s\in S}\{x : V^b(x,s) = V^D(s)\}$*, satisfying* $x^{lb} \leq x^{ub}$*.*

The lower bound of the ergodic set is determined by the lowest achievable relative Pareto weight in the contract. It represents the lowest value that the borrower accepts in the contract, which keeps it away from immiseration. The upper bound represents the highest relative Pareto weight that makes the borrower's constraint bind; therefore it is the highest weight that the lender may need to accept.[7]

Using Definition A.1, we can determine subprogram lengths in steady state. For this define $x_l(s) = \min\{x : \nu_l(x,s) > 0\}$ as the relative Pareto weight for which the limited enforcement constraint of the lender binds in s.

Lemma A.1 (Subprogram Length) *The steady state is such that the subprogram resets every time* $s' > s$*. In addition, it resets every time* $s' < s$ *if* $x_l(s') \in X^{ss}$*. Finally, for* $\eta < \tilde{\eta}$*, the subprogram resets more often with* η *than with* $\tilde{\eta}$*.*

The lemma is made of three parts. First, the subprogram resets as productivity improves. This is due to the binding constraint of the borrower coupled with the fact that $\eta < 1$. Second, the subprogram may not necessarily reset as productivity declines. This depends on whether the lender's constraint binds in steady state. More precisely, the subprogram resets as production declines only if this productivity state is associated to a binding constraint of the lender. Finally, the programs' length reduces as the borrower becomes more impatient.

Lemma A.2 (Subprogram Consumption) *Within a subprogram, consumption,* $c(\overline{x}, s)$*, decays at rate* η*. When a subprogram resets, consumption jumps up if* $x > \overline{x}$ *and jumps down if* $x < \overline{x}$*.*

[7] It should be noted that if the sovereign and the Fund are equally patient (i.e. $\eta = 1$), then the upper bound would be determined by $\min_{s\in S}\{x : V^l(x,s) = Z(s)\}$.

The lemma states that within a subprogram, there is perfect risk sharing adjusted for the relative impatience of the borrower η. Conversely, when a subprogram resets, consumption is adjusted up if the latent multiplier is larger than $\overline{x}$. In other words, consumption increases if the history of past shocks is associated with a large marginal cost of effort. The opposite is true when $x < \overline{x}$.

Lemma A.3 (Subprogram Labor) *Within a subprogram, labor, $n(\overline{x}, s)$, increases at rate η whenver ζ remains fixed. Otherwise, labor increases in ζ. When a subprogram resets, labor jumps down if $x > \overline{x}$ and jumps up if $x < \overline{x}$.*

Unlike consumption, labor is affected by changes in ζ within the subprogram as we can see in (12). Other than that labor follows the inverse pattern of consumption.

We can add more structure on subprograms. As we have seen before, the length of a subprogram is determined by the binding enforcement constraints. As these constraints bind according to the realization of s, the length of the subprogram is stochastic. However, as long as none of the enforcement constraints are binding, we can add additional (exogenous) criteria determining the length of a program. We consider a stochastic and a deterministic criteria.

Besides limited enforcement, consider that with exogenous probability χ a subprogram resets, while with probability $1-\chi$ the subprogram continues. In this case, the recursive SPFE reads,

$$
\begin{aligned}
\hat{FV}(x,s) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ & x\big[(1+\nu_b)(u(c)+h(1-n)-\hat{v}(e)) - \nu_b V^0(s) - \varrho v'(e)\big] \\
& + [(1+\nu_l)(\theta f(n)-c) - \nu_l Z] \\
& + \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s');\chi\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s');\chi\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x'(s') = \eta x \frac{1+\nu_b+\psi(s'\mid s,e)}{1+\nu_l} \\
& \bar{x}' = \eta x \frac{1+\nu_b}{1+\nu_l}
\end{aligned}
$$

where $\mathbb{I}_{\{(x',s');\chi\}} = \mathbb{I}_{\{(x',s')\}}(1-\chi)+\chi$ is such that the subprogram resets whenever one of the limited enforcement constraints is binding or resets with probability χ when no such constraints bind. Then within the subprogram

$$
\begin{aligned}
\overline{FV}(x,s;\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ & \bar{x}\left[u(c)+h(1-n)\right] - x\big[\hat{v}(e)+\varrho v'(e)\big] + (\theta f(n)-c) \\
& + \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s');\chi\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s');\chi\}})\overline{FV}(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x' = \eta x\left[1+\psi(s'\mid s,e)\right] \\
& \bar{x}' = \eta\bar{x},
\end{aligned}
$$

Except for the definition of the indicator function $\mathbb{I}_{\{(x',s');\chi\}}$, the problem is the same as the one considered in the previous section.

Instead of the stochastic component χ, we can add a deterministic component besides the limited enforcement constraint. In particular, suppose that the subprogram resets after a deterministic number of periods m if none of the limited enforcement constraints have bound until then. Starting with $\hat{FV}(x,s)$ for the start of the subprogram,

$$\begin{aligned}
\hat{FV}(x,s) = \min_{\{\nu_b,\nu_l,\varrho\}} \max_{\{c,n,e\}} \Big\{ & x\big[(1+\nu_b)(u(c)+h(1-n)-\hat{v}(e)) - \nu_b V^0(s) - \varrho v'(e)\big] \\
& + [(1+\nu_l)(\theta f(n) - c) - \nu_l Z] \\
& + \frac{1+\nu_l}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s')\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s')\}})\overline{FV}_1(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x'(s') = \eta x \frac{1+\nu_b+\psi(s' \mid s,e)}{1+\nu_l} \\
& \bar{x}' = \eta x \frac{1+\nu_b}{1+\nu_l}
\end{aligned}$$

then, for $k = 1, ..., m-1$,

$$\begin{aligned}
\overline{FV}_k(x,s;\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ & \bar{x}\left[u(c)+h(1-n)\right] - x\big[\hat{v}(e) + \varrho v'(e)\big] + (\theta f(n) - c) \\
& + \frac{1}{1+r}\mathbb{E}\left[\mathbb{I}_{\{(x',s');\chi\}}\hat{FV}(x',s') + (1-\mathbb{I}_{\{(x',s');\chi\}})\overline{FV}_{k+1}(x',s';\bar{x}') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x' = \eta x\left[1+\psi(s' \mid s,e)\right] \\
& \bar{x}' = \eta \bar{x},
\end{aligned}$$

and finally for m,

$$\begin{aligned}
\overline{FV}_k(x,s;\bar{x}) = \min_{\{\varrho\}} \max_{\{c,n,e\}} \Big\{ & \bar{x}\left[u(c)+h(1-n)\right] - x\big[\hat{v}(e) + \varrho v'(e)\big] + (\theta f(n) - c) + \frac{1}{1+r}\mathbb{E}\left[\hat{FV}(x',s') \mid s,e\right] \Big\} \\
\text{s.t.} \quad & x' = \eta x\left[1+\psi(s' \mid s,e)\right]
\end{aligned}$$

Obviously, the subprogram resets in m only if none of the limited enforcement constraint binds in all the m periods of the subprogram. Given this, the characterization is the same as before except that in period m,

$$\begin{aligned}
v'(e(x,s)) =& \beta \sum_{s'|s} \partial_e \Upsilon(s'|s,e)\hat{V}^b(x',s') + \frac{1+\nu_l}{1+\nu_b}\frac{1}{x}\frac{1}{1+r}\sum_{s'|s}\partial_e\Upsilon(s'|s,e)\hat{V}^b(x',s') \\
& - \varrho(x,s)\left[v''(e(x,s)) + \beta\sum_{s'|s}\partial_e^2\Upsilon(s'|s,e)\hat{V}^b(x',s')\right].
\end{aligned}$$

For any period $t \neq m$, the first-order condition is as the one derived previously.

B Proofs

B.1 Proof of Lemma 1

This is straightforward as larger ζ involves greater consumption. The rest follows the standard argument about the monotonicity of V^D.

B.2 Proof of Proposition 1

As shown by Ábrahám et al. (2025), (16) is concave under Assumptions 1-3 and the fact the the third derivative of v is zero. We can therefore apply Corollary 3 in Georgiadis et al. (2024) stating that the distribution has at most one g' in its support. As a result, we can reformulate (16) as a problem of choosing g' directly instead of $\pi^{g\prime}$.

B.3 Proof of Lemma 2

From (13), when $g' = \underline{g}$, $V^l(x', s') = V_0^l(x', \zeta')$ meaning that the FOC is satisfied only if $\varrho_{\pi^g(\zeta')}(s') = 0$. In opposition, when $g < \underline{g}$, $V^l(x', s') > V_0^l(x', \zeta')$ meaning that the FOC is satisfied only if $\varrho_{\pi^g(\zeta')}(s') > 0$.

B.4 Proof of Corollary 1

When $\eta = 1$ and $\nu_{l,t}(s^t) = 0$ in all states, the law of motion of the relative Pareto weight simplifies to the following expression

$$\mathbb{E}_t x_{t+1}(s^{t+1}) \equiv \mathbb{E}_t \left[\bar{x}_{t+1}(s^t) + \hat{x}_{t+1}(s^{t+1})\right] = \mathbb{E}_t \left[(1 + \nu_{b,t}(s^t))x(s^t) + \varrho_{\pi^g_{t+1}}(s^{t+1}|s^t)x(s^t)\right].$$

Since $(\nu_{b,t}(s^t), \varrho_{\pi^g_{t+1}}(s^{t+1}|s^t)) \geq 0$, we get that $\mathbb{E}_t x_{t+1}(s^{t+1}) \geq x_t(s^t)$.

B.5 Proof of Lemma 3

To see *i)*, note that, given the above decompostion of the law-of-motion of x, $\sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t, e(s^t))\hat{x}_{t+1}(s^{t+1}) = 0$, since independently of effort $\sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t, e(s^t)) = 1$, hence $\sum_{s^{t+1}|s^t} \partial_e \pi(s^{t+1}|s^t, e(s^t)) = 0$. Therefore $\mathbb{E}_t \hat{x}_{t+1} = 0$ and $\mathbb{E}_t x_{t+1} = \bar{x}_{t+1}(s^t)$. Alternatively, the expected law of motion of x can also be expressed as

$$\mathbb{E}_t x_{t+1} = \mathbb{E}_t \left[\frac{1}{u'(c_{t+1})} \frac{1 + \nu_{l,t+1}(s^t)}{1 + \nu_{b,t+1}(s^t)}\right] = \frac{1}{u'(c_t)}\eta,$$

where the last equality is the *inverse Euler equation* of the recursive contract (Ábrahám et al. (2025), Lemma 4).

To see *ii)*, note that, since the limited enforcement multipliers are either zero or at most one of the two is positive, we can have the following decomposition

$$\mathbb{E}_t \frac{1}{u'(c_{t+1})} = \mathbb{E}_t \left[x_{t+1} \frac{1 + \nu_{b,t+1}(s^{t+1})}{1 + \nu_{l,t+1}(s^{t+1})}\right] = \mathbb{E}_t x_{t+1} + \mathbb{E}_t x_{t+1}\nu_{b,t+1}(s^t) - \mathbb{E}_t x_{t+1} \frac{\nu_{l,t+1}(s^t)}{1 + \nu_{l,t+1}(s^t)},$$

where $\mathbb{E}_t x_{t+1} = \eta x_t$ and, without incentive constraints, the last two terms simply denote the change in the relative Pareto weight when either the no-default or the sustainability constraints binds. However, if limited enforcement constraints satisfy the ‘no-free-lunch condition’, the no-default constraint is more likely to bind, while the sustainabiliy constraint is less likely to bind and, as a result, in both cases expected consumption increases.