Efficient Sovereign Debt Buybacks*

Adrien Wicht
University of Basel

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Abstract

This paper explores the conditions under which sovereign debt buybacks are Pareto efficient, challenging the conventional view that such operations are detrimental to sovereign borrowers. Using a model of strategic lending, I show that buybacks can be rationalized as part of an optimal contract between a sovereign borrower and foreign lenders. In particular, buybacks allow bonds to function like Arrow securities. This is because they take place in the secondary market, where only legacy lenders operate, granting them market power, as opposed to the primary market, where new entrants ensure competitive returns. The model aligns with the recent empirical evidence in Brazil, including a premium paid on buyback operations. These findings offer insights into sovereign debt management and the implementation of optimal contracts.

Keywords: sovereign debt, buyback, constrained Pareto efficiency, strategic lending
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Correspondence: Adrien Wicht, University of Basel, Faculty of Business and Economics, Peter-Merian Weg 6, 4002 Basel, Switzerland. E-mail: adrien.wicht@unibas.ch.

1 Introduction

Countries regularly engage in the repurchase of previously issued bonds, a process known as a buyback. Following the seminal contribution of Bulow and Rogoff (1988, 1991), the literature on sovereign debt and default generally considers buybacks to be inefficient. The reason is that such operations are detrimental to borrowers. The purpose of this paper is to show that there are conditions under which buybacks can be Pareto efficient. I first expose such conditions in a model of strategic lending and then relate them to the recent empirical evidence in Brazil.

I show that the main aspects of sovereign debt buybacks can be rationalized as part of an optimal contract between a risk-averse sovereign borrower and risk-neutral foreign lenders. In particular, buybacks provide a clear interpretation for the binding participation constraint in optimal contracts under limited commitment. These contracts exhibit the characteristic that the borrower's participation constraint binds when the endowment increases. High endowment states are also associated with lower levels of indebtedness and capital outflows, aligning with the idea of a buyback.

More generally, buybacks allow bonds to function like Arrow securities. This is because issuances and buybacks occur in two distinct markets, creating state contingency through different payouts in these markets. More precisely, I distinguish between the primary market, where new bonds are issued, and the secondary market, where existing bonds can be repurchased before maturity. The crux of this distinction is that buybacks take place in the secondary market, where only legacy lenders operate, granting them market power. In contrast, new issuances occur in the primary market, where new entrants ensure competitive returns. Hence, legacy long-term debt yields higher payouts in the secondary market than in the primary market. The recent buybacks in Brazil provide supportive evidence of that.

I consider an endowment economy with one sovereign borrower and *two* foreign lenders. Endowment is stochastic with two states: high and low. The lenders are risk neutral and trade non-contingent bonds of different maturities with the borrower. Only one of the two lenders holds the outstanding bonds every period. I call it the incumbent lender which represents the entire group of legacy lenders. The other lender is the outsider which represents new entrants. The borrower is risk averse, impatient and lacks commitment.

I first characterize the equilibrium in the market economy. In the primary market, the two lenders compete with each other, offering prices and quantities that ensure zero expected profit. Offers must also respect a cross-lender seniority clause protecting the incumbent lender from dilution. This limits the quantity of bonds the borrower can issue overall. In the secondary market, the incumbent lender is the sole holder of legacy long-term debt. It uses its market power to offer buybacks at a premium that makes the borrower indifferent between repurchasing debt and defaulting. Similar to Müller et al. (2019), for the market outcome to be efficient, the incumbent lender possesses all the market power *ex post*.

The assumption of senior legacy long-term debt is critical. In fact, the model becomes isomorphic to the canonical sovereign default model without it. With long-term debt, the incumbent lender is unwilling to offer bonds which dilute legacy debt as dilution directly reduces its payoff. With seniority, the outsider internalizes this dilution aversion. If one removes long-term debt, there is no secondary market. There is also no dilution of legacy debt which makes the two lenders indifferent to the default risk. If one removes seniority, the outsider satiates the borrower's demand for bonds without caring for dilution.

On the equilibrium path, buybacks and defaults are mutually exclusive. While defaults do not arise in the high endowment state, buybacks do not arise in the low endowment state. This is due to the strict concavity of the utility function. A default increases current consumption, while a buyback has the opposite effect. In the low endowment state, a reduction in consumption is too costly in terms of utility and overcomes the gains of higher future consumption. As a result, in the low endowment state, there is no buyback and, in the high endowment state, repayment is preferred towards default.

More broadly, the distinction between buybacks and defaults is inherent to the distinction between complete and incomplete markets. As noted by Arellano (2008), in incomplete markets, the surplus between the value of repayment and the value of autarky decreases when the endowment decreases and is i.i.d. As a result, the threat of autarky is higher in the low endowment state implying that defaults happen in bad times. In complete markets, the reverse holds as the threat of autarky fades in the low endowment state implying that buybacks arise in the high endowment state. Consequently, buybacks are the mirror image of the re-contracting framework of Bulow and Rogoff (1989) in which the notional payment is the highest and lower payments arise in the low endowment state through renegotiation. In my case, the notional payment arise in the low endowment state and a premium is paid in the high endowment state via the buyback.[1]

I then show that the market economy can implement the allocation of a Planner. The Planner allocates resources between the borrower and the lenders, accounting for a participation constraint to ensure the borrower receives at least the value of autarky. This defines a constrained efficient allocation which features risk sharing in the form of state-contingent debt re-valuation. In the high endowment state, the borrower's debt is re-valuated upward and consumption is strictly less than endowment indicating a capital outflow.

To implement the constrained efficient allocation, the borrower conducts buybacks in

[1] I thank Mark Aguiar for noticing the parallel.

the high endowment state. Due to the payment of the premium, the buyback increases the value of outstanding long-term debt resulting in a capital loss for the borrower. In the low endowment state, there is no buyback and therefore no premium paid. The opposite argument holds. Thus, the differentiated payout between the primary and the secondary market generates the capital losses and gains necessary to replicate the state contingency in the Planner's allocation. In steady state, buybacks occur whenever the endowment increases. In that sense, buybacks are frequent. In opposition, defaults never arise owing to the presence of senior legacy long-term debt.

I relate my findings to the experience of Brazil from 2006 to 2019. The country defaulted last in 1989 and regained access to the international bond market with a Brady Plan in 1994. In 2006, it started a comprehensive buyback program which aimed at correcting the average maturity and reducing the refinancing risk. Focusing on the repurchase of USD-denominated bonds, I find that the dynamic of the Brazilian buybacks is in line with the model's prediction. Buybacks led to a substantial reduction in the weighted average maturity and in indebtedness. They mainly occurred when output was growing and the interest rate spread was low. This is a stark contrast with buybacks documented by Bulow and Rogoff (1988) in the 1980s which occurred when the default risk was high.

I estimate the premium paid by the Brazilian government during buyback operations. Building on the measure of creditor loss of Sturzenegger and Zettelmeyer (2008), my approach consists of reconstructing the stream of cashflows of the bonds involved in buyback operations. In particular, I compare the present value of a bond with and without the buyback at the issue yield to maturity. To ensure that such yield does not anticipate future buybacks, I restrict my attention to bonds issued prior to the start of the Brazilian buyback program in 2006. I find that the premium averages 14.94%. It is due to the fact that bonds were mostly issued below the par value and bought back above the par value.

The paper is organized as follows. Section 1.1 reviews the related literature. Section 2 lays down the model environment. Sections 3 and 4 expose the market and the Planner economy, respectively. Section 5 presents the quantitative results. Section 6 concludes. The Online Appendix contains additional results, data sources and proofs.

1.1 Related literature

The paper contributes to the literature on sovereign defaults. It builds on the two-lender-one-borrower market structure of Kovrijnykh and Szentes (2007) with the standard timing of actions of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). Similar to Arellano and Ramanarayanan (2012) and Niepelt (2014), I adopt two bonds with different maturities. Moreover, I distinguish between the primary and the secondary bond markets as in Broner et al. (2010). I contribute to this literature in two ways. First, I depart from the standard assumption of competitive lending by modelling lenders with market power in the secondary market. This differs from Bi (2008), Yue (2010) and Benjamin and Wright (2013) who consider bargaining power during debt restructurings. Second, I analyze the consequences of long-term debt dilution in strategic lending. This complements the analysis of Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Chatterjee and Eyigungor (2015) focusing on competitive bond markets.

This paper focuses on sovereign debt buybacks and relates to the seminal contribution of Bulow and Rogoff (1988, 1991) who document that such operations are detrimental to borrowers since the reduction in future payments is less in expected value than the repurchase price. The rationale is that buybacks increase the recovery value per unit of bond given a fixed collateral value upon default. In that logic, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Without collateral value upon default, Aguiar et al. (2019) also find that buybacks reduce welfare as they shift the maturity structure and therefore affect the default risk. I challenge this view by establishing the Pareto efficiency of buybacks as a long-term risk sharing agreement between the borrower and the lenders. Rotemberg (1991) and Acharya and Diwan (1993) also highlight potential benefits of buybacks, though from different angles. The former studies bargaining costs, while the latter examines the signalling effect of buybacks. More importantly, like Bulow and Rogoff (1988, 1991), both studies primarily focus on a state of debt overhang, whereas I consider the full equilibrium dynamic in evaluating welfare. Additionally, I estimate buyback premia in the data adapting the framework of Sturzenegger and Zettelmeyer (2008).

The paper addresses the literature on optimal contracts and their implementation. Deriving a contract between a borrower with limited commitment and foreign lenders, my study relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). Similar to Kehoe and Perri (2002) and Restrepo-Echavarria (2019), I use the Lagrangian approach of Marcet and Marimon (2019) with the difference that I implement the optimal contract in a market economy.

My implementation is the closest to Dovis (2019). The main difference is that the author considers a production economy with privately-observed productivity shocks. His decentralization relies on defaults and non-contingent bonds of different maturities building on the concept of excusable defaults in Grossman and Van Huyck (1988) and the maturity management exposed in Angeletos (2002) and Buera and Nicolini (2004). The state contingency of the bond contract originates from defaults in bad times rather than buybacks in good times. Moreover, the enforcement of the default policy emanates from trigger strategies instead of

Markov strategies.

Besides this, Alvarez and Jermann (2000) implement the allocation in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. In my implementation, long-term debt behaves like an Arrow security owing to the different payout between the primary and the secondary markets. There is also a borrowing limit which emanates from the seniority of legacy long-term debt. Thus, the main difference with my analysis is that the two authors assume a greater financial sophistication as securities are state contingent, while I rely on seniority and market power of the legacy lenders.

In addition, Müller et al. (2019) propose an implementation in an environment with a stochastic default cost and two endowment states. The authors assume a financial market formed by a one-period defaultable bond and a endowment-contingent bond. The defaultable bond spans the different stochastic default costs through renegotiation, while the contingent bond spans the endowment states. The difference is that my implementation relies on an active debt maturity management on the primary and the secondary bond markets with a fixed default cost.

In a similar environment, Aguiar et al. (2019) decentralize a constrained efficient allocation using a continuum of maturities.[2] They consider a Planner's problem with no participation constraint. This results in a constrained efficient allocation consistent with incomplete markets, while my allocation is consistent with complete markets. Moreover, the Planner does not take into consideration the legacy lenders in the surplus maximization, while such lenders represent the main actor in my environment. The reason is that the authors assume a competitive lending market without seniority. Absent any market power, the legacy lenders are unable to extract surplus from the borrower. Consequently, the market economy cannot achieve overall efficiency for all contracting parties.

2 Environment

This section presents the different market participants, the organisation of the international bond market and the timing of actions.

2.1 Market participants

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The government of this economy (i.e. the borrower) is benevolent, receives an endowment every period and can trade bonds with two foreign lenders.

[2] See also the textbook treatment in Aguiar and Amador (2021).

Endowment is stochastic. It takes value on the discrete set $Y \equiv \{y_L, y_H\}$ with $0 < y_L < y_H$ and is independent and identically distributed (i.i.d) with $\pi(y_{t+1})$ corresponding to the probability of drawing y_{t+1} at date $t+1$.

The borrower is impatient and discounts the future at rate $\beta < \frac{1}{1+r}$ with r being the exogenous risk-free rate. Preference over consumption is represented by $\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t)$ where $c_t \geq 0$ denotes the consumption at time t and $\mathbb{E}(\cdot)$ denotes the expectation. The instantaneous utility function $u(\cdot)$ is continuous, strictly increasing, strictly concave and satisfies the Inada condition $\lim_{c\to 0} u_c(c) = \infty$ where $u_c(\cdot)$ denotes the first derivative of $u(\cdot)$.

The borrower cannot commit to repay the lenders. In case of default, the borrower suffers from permanent autarky. There is no endowment penalty upon default and the lenders are able to seize the borrower's assets, if any.

The two foreign lenders are risk neutral and discount the future at rate $\frac{1}{1+r}$. As one will see, only one of the two lenders effectively holds bonds in a given period. This is the sole difference between the two lenders.

2.2 Bond markets

The borrower can trade bond contracts with two different maturities: short-term and long-term.[3] The short-term bond b'_{st} has a unit price q_{st}, matures in the next period and pays a coupon of one. Following Hatchondo and Martinez (2009), the long-term bond b'_{lt} has a unit price q_{lt}, matures at a rate $(1-\delta) \in [0, 1)$ and pays a coupon of one every period. The risk-free return is given by $\overline{q} = \frac{1}{1+r-\delta}$. I denote debt as a negative asset meaning that $b'_j < 0$ is a debt, while $b'_j > 0$ is an asset for all $j \in \{st, lt\}$. The borrower is assumed to always be a net debtor, i.e. $b_{st} + b_{lt}(1 + \delta q_{lt}) \leq 0$.

I distinguish between the primary and the secondary markets for bond contracts. In the former new bond contracts are issued, while in the latter (part of) perviously-issued bond contracts can be retired. Operations on the secondary market are called *buybacks* and are relevant only for long-term bond contracts since short-term bond contracts last one period. If the borrower enters the secondary market for long-term bond contracts, it can access the primary market for short-term bond contracts. Thus, the borrower decides to enter in either the primary market or both markets simultaneously given (y, b_{st}, b_{lt}).

If the borrower enters the primary market only, the two lenders simultaneously offer a pair of short-term and long-term bond contracts. The borrower can only accept one of the two offers. Similar to Kovrijnykh and Szentes (2007), I call a lender the *incumbent* at the beginning of a period if the borrower accepted its offer in the previous period. The other

[3] To decentralize the constrained efficient, I need as many maturities as endowment states.

lender is called the *outsider*. I denote by $(b_{st}^{o\prime}, q_{st}^{o}; b_{lt}^{o\prime}, q_{lt}^{o})$ the offer made by the outsider and by $(b_{st}^{i\prime}, q_{st}^{i}; b_{lt}^{i\prime}, q_{lt}^{i})$ the offer made by the incumbent. If the borrower accepts the outsider's offer, the outsider takes over the long-term bond contract from the incumbent. For given outstanding bonds (b_{st}, b_{lt}), transfers are as follows. The borrower receives $-q_{st}^{o}b_{st}^{o\prime} - q_{lt}^{o}(b_{lt}^{o\prime} - \delta b_{lt})$ from the outsider and pays $b_{st} + b_{lt}$ to the incumbent. The outsider then pays $-q_{lt}^{m}\delta b_{lt}$ to the incumbent where $q_{lt}^{m} = q_{lt}^{o} + \max\{q_{lt}^{i} - q_{lt}^{o}, 0\}\mathbb{1}_{\{b_{lt}<0\}}$ and $\mathbb{1}_{\{x\}}$ is an indicator function taking value one if x is true.

If the borrower enters both the primary and the secondary markets, the outsider and the incumbent simultaneously offer a short-term bond contract in the primary market. In the secondary market, the incumbent makes a take-it-or-leave-it offer $(b_{lt}^{i\prime}, q_{lt}^{i}, \chi)$ which consists of a debt buyback $b_{lt}^{i\prime} - \delta b_{lt} < 0$ with $b_{lt} < 0$ and a buyback premium $\chi > 0$. Given this, when the incumbent offers $(b_{st}^{i\prime}, q_{st}^{i}; b_{lt}^{i\prime}, q_{lt}^{i}, \chi)$ and the outsider offers $(b_{st}^{o\prime}, q_{st}^{o})$, then by a slight abuse of notation, one can say that the outsider effectively offers $(b_{st}^{o\prime}, q_{st}^{o}; b_{lt}^{i\prime}, q_{lt}^{i}, \chi)$. If the outsider's offer is accepted, the outsider takes over the long-term bond contract from the incumbent as before. For given outstanding bonds (b_{st}, b_{lt}), the borrower receives $-q_{st}^{o}b_{st}^{o\prime}$ from the outsider and pays $-q_{lt}^{i}(b_{lt}^{i\prime} - \delta b_{lt}) + b_{st} + b_{lt}(1 + \delta\chi)$ to the incumbent. The outsider then receives $q_{lt}^{i}b_{lt}^{i\prime}$ from the incumbent.

Every period there is only one of the two lenders holding all the bond contracts. Hence, the incumbent can be interpreted as the entire group of legacy lenders. This supposes that this specific group of lenders can coordinate their actions. In opposition, the outsider corresponds to the entire group of lenders which hold no claim on the borrower. It reflects new entrants in the international bond market.

In the transfer of long-term bond contracts, the assumption that the outsider repays at least the price offered by the incumbent implies seniority of legacy long-term debt. If there is no default risk, this assumption is without loss of generality. However, if there is a risk of default, the seniority prevents the outsider to dilute legacy claims. The seniority of legacy debt therefore makes the outsider internalize the cost of dilution. Note that the seniority solely applies to long-term debt, i.e. $b_{lt} < 0$.

2.3 Timing of actions

I consider Markov equilibria. That is I restrict my attention to the payoff-relevant state vector $\Omega \equiv (y, b_{st}, b_{lt})$.[4] The timing of actions is the same as in Eaton and Gersovitz (1981). At the beginning of each period, y realizes and the borrower decides whether to default given

[4]To save up notation, I ignore the indicator function taking value one when the borrower is in default. Strictly speaking such variable should also be part of the state vector.

(b_{st}, b_{lt}). Conditional on no default, the borrower decides to enter either the primary market only (i.e. no buyback) or both the primary and the secondary markets (i.e. buyback).

The timing of actions diverges from the one of Kovrijnykh and Szentes (2007) in which the lenders first offer bond contracts – stating a current payment and a value of the bond next period – and then the borrower decides whether to default. In my case, the default decision happens before offers are made and is taken as given by the two lenders. Furthermore, the bond contract specifies the amount lent (i.e. the price times the quantity of bonds) and the payment (i.e. the coupon times the quantity of bonds) for the next period.

3 Market Economy

This section exposes the market economy. It first derives the borrower's and the lenders' problems and subsequently characterizes the underlying equilibrium.

3.1 Borrower problem

The borrower's overall beginning of the period value is

$$V(\Omega) = \max \left\{ V_B^P(\Omega), V_{NB}^P(\Omega), V^D(y) \right\}, \tag{1}$$

where $V_B^P(\cdot)$ and $V_{NB}^P(\cdot)$ correspond to the value of repayment in case of buyback and no-buyback, respectively, and $V^D(\cdot)$ corresponds to the value of default.

Under repayment without buyback, the borrower enters the primary market only and chooses the offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ which maximizes its value subject to the budget constraint. Formally, one has that

$$\begin{aligned} V_{NB}^P(\Omega) = & \max_{(b'_{st}, q_{st}; b'_{lt}, q_{lt}) \in \Gamma} \left\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \right\} \\ & \text{s.t.} \quad c + q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}. \end{aligned} \tag{2}$$

The set Γ contains the offers of the two lenders. Given that there is no debt buyback, all offers in Γ are such that $b'_{lt} \leq \delta b_{lt}$ when $b_{lt} < 0$.

Under repayment with a buyback, the borrower enters both the primary market to issue new short-term bond contracts and the secondary market to repurchase existing long-term bond contracts. Formally,

$$V_B^P(\Omega) = \max_{(b'_{st}, q_{st}; b'_{lt}, q_{lt}, \chi) \in \Lambda} \left\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \right\} \tag{3}$$

$$
\text{s.t.} \quad c + q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1 + \delta\chi),
$$
$$
b_{lt} < 0.
$$

The set Λ contains the two offers for the short-term bond contract each combined with the incumbent's offer for the long-term bond contract. The incumbent's buyback offer is such that $\chi > 0$ and $b'_{lt} > \delta b_{lt}$ with $b_{lt} < 0$. If $b_{lt} \geq 0$, there is no possibility to buyback and $V_B^P(\Omega) = -\infty$. Finally, the value under default is given by

$$
V^D(y) = u(y) + \beta \mathbb{E}\left[V^D(y')\right]. \tag{4}
$$

The assumption that assets can be seized is to ensure that the value of default is independent of (b_{st}, b_{lt}). Recall that I assume that the borrower is a net debtor in every state.

Given this, I can define $D(\Omega)$ as the default policy which takes value 1 if $V^D(y) > \max\{V_B^P(\Omega), V_{NB}^P(\Omega)\}$ and zero otherwise. Similarly, $M(\Omega)$ is the buyback policy which takes value 1 if $V_B^P(\Omega) > \max\{V_{NB}^P(\Omega), V^D(y)\}$ and zero otherwise. If the borrower is indifferent between a default and a buyback, it chooses the latter. Finally, $B_{st}(\Omega) = b'_{st}$ and $B_{lt}(\Omega) = b'_{lt}$ correspond to the short-term and long-term bond policy, respectively.

3.2 Lenders problem

Looking at the lenders' problem, I can specify the offer sets Γ and Λ. I first consider the offers on the primary market before turning to the secondary market.

In the primary market, the outsider is willing to offer bond contracts under which its expected payoff is large enough to cover the transfer to the incumbent. Define the value of the incumbent in state Ω by $W(\Omega)$. If the borrower enters the primary market only, the outsider's offer $(b^{o\prime}_{st}, q^o_{st}; b^{o\prime}_{lt}, q^o_{lt})$ has to be such that

$$
q^o_{st} b^{o\prime}_{st} + q^o_{lt}(b^{o\prime}_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}\left[W(y', b^{o\prime}_{st}, b^{o\prime}_{lt})\right] \geq -q^m_{lt}\delta b_{lt}. \tag{5}
$$

If the outsider's offer satisfies this constraint, the incumbent would offer the same contract. The reason is as follows. If the incumbent makes the same offer and it is accepted, the incumbent receives $-(b_{st} + b_{lt})$ plus the left-hand side of (5). Otherwise, the incumbent receives $-(b_{st} + b_{lt})$ plus the right-hand side of (5).

Similarly, if the borrower enters both the primary and the secondary markets, the outsider's offer $(b^{o\prime}_{st}, q^o_{st})$ has to be such that

$$
q^o_{st} b^{o\prime}_{st} + q^i_{lt} b^{i\prime}_{lt} + \frac{1}{1+r}\mathbb{E}\left[W(y', b^{o\prime}_{st}, b^{i\prime}_{lt})\right] \geq 0. \tag{6}
$$

One can repeat the same argument as before. If the incumbent makes the same offer as the outsider and it is accepted, the incumbent receives $-(b_{st}+b_{lt}(1+\delta\chi+\delta q^i_{lt}))$ plus the left-hand side of (6). Otherwise, the incumbent receives $-(b_{st}+b_{lt}(1+\delta\chi+\delta q^i_{lt}))$. Hence, if the outsider's offer satisfies (6), the incumbent would offer the same contract. Without loss of generality, one can assume that the incumbent's offer in the primary market is always the one accepted by the borrower on the equilibrium path.

In the primary market, the two lenders compete with each other. This means that they both record zero expected profit. If one of the two lenders makes an offer leading to a strictly positive expected profit, the other lender can undercut this offer. As a result, prices are competitive in the primary market. For the short-term bond price, this means that

$$q_{st}(b'_{st},b'_{lt}) = \begin{cases} \frac{1}{1+r}\mathbb{E}\Big[(1-D(\Omega'))+D(\Omega')R_{st}(\Omega')\Big] & \text{if } b'_{st}<0 \\ \frac{1}{1+r} & \text{else} \end{cases} \tag{7}$$

where $R_{st}(\Omega') \equiv \frac{1}{1+r}\frac{\max\{0,(1+r)q_{lt}(b'_{st},b'_{lt})b'_{lt}\}}{-b'_{st}}$ represents the recovery value upon default. Since assets can be seized during defaults, the recovery value can be strictly positive. If $b'_{st}\geq 0$, the short-term bond price equates the risk-free return as lenders do not default.[5] If $b'_{st}<0$, the short-term bond price accounts for the default risk. Note that the bond price is independent of y given the i.i.d assumption. Conversely, the long-term bond price is given by

$$q_{lt}(b'_{st},b'_{lt}) = \begin{cases} \frac{1}{1+r}\mathbb{E}\Big[(1-D(\Omega'))\{Q_{lt}(\Omega')+M(\Omega')\delta\chi(\Omega')\}+D(\Omega')R_{lt}(\Omega')\Big] & \text{if } b'_{lt}<0 \\ \frac{1}{1+r}\mathbb{E}\Big[Q_{lt}(\Omega')\Big] & \text{else} \end{cases} \tag{8}$$

where $R_{lt}(\Omega') \equiv \frac{1}{1+r}\frac{\max\{0,b'_{st}\}}{-b'_{lt}}$ is the recovery value upon default and $Q_{lt}(\Omega')\equiv 1+\delta q_{lt}(s',b''_{st},b''_{lt})$ with $b''_{st}=B_{st}(\Omega')$ and $b''_{lt}=B_{lt}(\Omega')$ is the notional return upon repayment. If $b'_{lt}\geq 0$, the bond price is not always equal to $\overline{q}$. This is because the lenders offer one price and not two separate prices for b''_{lt} and $\delta b'_{lt}$. Such distinction only matters when there is a sign reversal between b''_{lt} and $\delta b'_{lt}$, though. Notice that δ is restricted to be strictly larger than zero, otherwise (7) and (8) would be identical.

Even though the two lenders always make zero expected profit, they do not offer all possible bond quantities in the primary market. In particular, the incumbent is not willing to offer bonds which would dilute legacy long-term debt. This is because under zero expected profit the incumbent's value corresponds to the value of outstanding bond contracts, $b_{st}+b_{lt}(1+\delta q^m_{lt})$. Hence, if $b_{lt}<0$, the incumbent chooses $(b^{i\prime}_{st},b^{i\prime}_{lt})$ such that the value of long-term

[5] Given that the borrower is a net debtor by assumption, the lenders are net creditors.

debt is maximized. The outsider has no incentive to make another offer as this would violate (5) given that $q^m_{lt} = \max\{q^o_{lt}(b^{o\prime}_{st}, b^{o\prime}_{lt}), q^i_{lt}(b^{i\prime}_{st}, b^{i\prime}_{lt})\}$ when $b_{lt} < 0$. In other words, the outsider's preference aligns with the incumbent's through the seniority of legacy debt. If $b_{lt} > 0$, this is not true anymore as the incumbent has a long-term debt towards the borrower and would like to dilute that claim.

Given the seniority of legacy long-term debt, the two lenders are not perfect Bertrand competitors in the primary market. While it is true that they offer prices leading to zero expected profit, they may restrict the quantity of bonds offered to the borrower.

In the secondary market, the incumbent is the sole holder of legacy long-term debt. It therefore acts as a monopolist for the repurchase of outstanding long-term bond contracts. I assume that the incumbent possesses all the market power *ex post*. Especially, the incumbent makes a take-it-or-leave-it offer such that the borrower is indifferent between repayment and default. Hence, the buyback premium is such that

$$\chi(\Omega) = \inf\{\chi > 0 : V^P_B(\Omega) = V^D(y)\}. \tag{9}$$

If there is no $\chi > 0$ in Ω such that $V^P_B(\Omega) = V^D(y)$, the offer is rejected (i.e. $\chi = \infty$). In opposition, if there exists a $\chi > 0$ making the borrower indifferent, the premium is *tight* in the sense of Alvarez and Jermann (2000). It corresponds to the highest possible premium the incumbent can charge consistent with no default.

Note that the market power of the incumbent solely pertains to the repurchase of the legacy debt δb_{lt}. In that logic the incumbent imposes a premium χ on δb_{lt} but still offers a new level of bond $b^{i\prime}_{lt}$ at a competitive price q^i_{lt}. If the incumbent makes a strictly positive expected profit on the new long-term bond, the outsider can offer a short-term bond contract at a loss which compensates the gain on the long-term bond contract such that (6) holds with equality. The borrower would prefer that offer and the outsider becomes the incumbent.

The following proposition summarizes the main characteristics of the lender's offer in both the primary and the secondary markets.

Proposition 1 (Optimal Offers)**.** *In the primary and the secondary markets, the two lenders offer bond contracts with prices (q_{st}, q_{lt}) satisfying (7)-(8) and quantities (b'_{st}, b'_{lt}) such that*

$$q_{lt}(b'_{st}, b'_{lt}) = \begin{cases} \arg\max_{\tilde{b}'_{st}, \tilde{b}'_{lt} \le \delta b_{lt}} \left\{ q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt}) \right\} & \text{if } b_{lt} < 0 \text{ and } b'_{lt} \le \delta b_{lt} \\ \arg\max_{\tilde{b}'_{st}, \tilde{b}'_{lt} > \delta b_{lt}} \left\{ q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt}) \right\} & \text{if } b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt} \end{cases} \tag{10}$$

implying that (5)-(6) hold with equality in all Ω.

Condition (10) together with competitive prices ensures that (5) and (6) always hold with equality. It is true that the incumbent can drive the outsider out of the primary market by making offers such that (5) or (6) cannot hold. Nevertheless, such offers lead to net losses. As a result, the incumbent never acts as a monopolist on the primary market. This the main difference with respect to Kovrijnykh and Szentes (2007). The reason is that the borrower decides to default before the bond auction and the lenders take this decision as given.

Given this, the offer set in the primary market is $\Gamma = \{(b'_{st}, q_{st}; b'_{lt}, q_{lt}) : (b'_{lt} - \delta b_{lt}) \leq 0 \text{ if } b_{lt} < 0 \wedge (7) \wedge (8) \wedge (10)\}$ and in the secondary market is $\Lambda = \{(b'_{st}, q_{st}; b'_{lt}, q_{lt}, \chi) : (b'_{lt} - \delta b_{lt}) > 0 \wedge (7)\text{-}(10)\}$.

3.3 Equilibrium properties

Combining the the previous two subsections together, I can reformulate the borrower's maximization problems in (2) and (3) accounting for the definition of the offer sets Γ and Λ. The problem is as if the borrower would offer bond contracts to the lenders subject to the constraint that bond prices are competitive and that the value of outstanding debt is maximized.

Proposition 2 (Optimal Bond Contracts)**.** *In equilibrium, if $M(\Omega) = 0$, the optimal bond contracts $B_{st}(\Omega)$ and $B_{lt}(\Omega)$ solve*

$$
\begin{aligned}
V^P_{NB}(\Omega) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \right\} \\
s.t. \quad & c + q_{st}(b'_{st}, b'_{lt}) b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}, \\
& b'_{lt} \leq \delta b_{lt} \text{ if } b_{lt} < 0. \\
& (7), (8) \text{ and } (10),
\end{aligned}
$$

and, if $M(\Omega) = 1$, the optimal bond contracts $B_{st}(\Omega)$ and $B_{lt}(\Omega)$ solve for a given χ

$$
\begin{aligned}
V^P_{B}(\Omega) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \right\} \\
s.t. \quad & c + q_{st}(b'_{st}, b'_{lt}) b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1 + \delta\chi), \\
& b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt}, \\
& (7), (8) \text{ and } (10),
\end{aligned}
$$

and $\chi(\Omega)$ satisfies (9).

The next proposition specifies the conditions under which defaults and buybacks do not occur. As a direct corollary of (10) the default risk diminishes when $b_{lt} < 0$. Moreover, defaults do not arise in the high endowment state, whereas buybacks do not arise in the low endowment state. This comes from the strict concavity of $u(\cdot)$. A default increases current consumption, while a buyback has the opposite effect. In y_L a reduction in consumption is too costly and overcomes the gains of higher future consumption. As a result, there is no buyback in y_L. Conversely, in y_H, repayment is less costly making default suboptimal.

Proposition 3 (Default and Buyback)**.**

I. (Long-Term Debt). *For any* $b_{lt} < 0$, $\mathbb{E}[D(y', B_{st}(\Omega), B_{lt}(\Omega))] \leq \mathbb{E}[D(\Omega)]$.

II. (Default). *For any* Ω, $D(y_H, B_{st}(\Omega), B_{lt}(\Omega)) = 0$.

III. (Buyback). *For any* (b_{st}, b_{lt}), $M(y_L, b_{st}, b_{lt}) = 0$.

The first part of the proposition is due to the seniority of legacy long-term debt. With long-term debt, dilution directly reduces the incumbent's payoff. With seniority, the outsider's offer must cover the transfer of legacy debt at no less than the incumbent's offered price. The two lenders are therefore averse to dilution. If one removes the long-term debt, there is no secondary market and no dilution which makes the two lenders indifferent to the default risk. Without seniority, the outsider is indifferent to the default risk and will always satiate the borrower's demand for bonds. As a result, the absence of both seniority and legacy long-term debt make the lenders always behave like perfect Bertrand competitors. In other words, the model becomes isomorphic to the canonical sovereign debt model.

The second and third parts of the proposition are inherent to the distinction between complete and incomplete markets. As noted by Arellano (2008), in incomplete markets, $V^P_{NB}(\Omega) - V^D(y)$ is increasing in y. As a result, the threat of autarky is higher in the low endowment state implying that defaults happen in bad times. In opposition, in complete markets, $V^P_{NB}(\Omega) - V^D(y)$ is decreasing in y. Consequently, the threat of autarky fades in the low endowment state. Equation (9) combined with the fact that buybacks cannot occur in y_L make buybacks feasible only in y_H and when markets are complete.

I end this section with the fact that the equilibrium is not unique. As in Alvarez and Jermann (2000), one cannot rule out permanent autarky as an equilibrium as shown in the following lemma

Lemma 1 (Permanent autarky)**.** *Permanent autarky is an equilibrium.*

The proof relies on the assumption that the borrower is a net debtor in any state. If the lenders believe that the borrower will default in every state and set bond prices accordingly, the borrower finds optimal to choose $D(\Omega) = 1$ for all Ω confirming the lenders' beliefs.

4 Planner Economy

This section derives the constrained efficient allocation. I first characterize the Planner problem and subsequently decentralize the underlying allocation in the market economy.

4.1 Planner's problem

The constrained efficient allocation is the outcome of a problem in which a Planner allocates consumption to maximize the lenders' and the borrower's weighted utility subject to a participation constraint. The participation constraint accounts for limited commitment in repayment (Thomas and Worrall, 1994). Denoting y^t as the history of realized endowment at time t, it must hold that for all t and y^t

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(c(y^j)) \geq V^D(y_t). \tag{11}$$

This condition ensures that the borrower's value is at least as large as the value of autarky. Given this, the Planner's maximization problem in sequential form reads

$$\max_{\{c(y^t)\}_{t=0}^{\infty}} \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) u(c(y^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{y^t} \pi(y^t) \left[y_t - c(y^t)\right] \tag{12}$$
$$\text{s.t.} \quad (11) \text{ for all } y^t, t \text{ with } (\mu_{b,0}, \mu_{l,0}) > 0 \text{ given.}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial Pareto weights assigned by the Planner to the borrower and the lenders, respectively. Following Marcet and Marimon (2019), I reformulate (12) as a saddle-point Lagrangian problem,

$$\mathcal{SP} \quad \min_{\{\gamma(y^t)\}_{t=0}^{\infty}} \max_{\{c(y^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) \mu_{b,t}(y^t) u(c(y^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{y^t} \pi(y^t) \mu_{l,t}(y^t) \left[y_t - c(y^t)\right]$$
$$+ \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) \gamma(y^t) \Big[\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(c(y^j)) - V^D(y_t) \Big]$$
$$\mu_{b,t+1}(y^t) = \mu_{b,t}(y^t) + \gamma(y^t) \text{ and } \mu_{l,t+1}(y^t) = \mu_{l,t}(y^t) \text{ for all } y^t, t$$
$$\text{with } \mu_{b,0}(y_0) \equiv \mu_{b,0} \text{ and } \mu_{l,0}(y_0) \equiv \mu_{l,0} \text{ given.}$$

In this formulation, $\gamma(y^t)$ denotes the Lagrange multiplier attached to the participation constraint at time t. As the value of the borrower appears in both the Planner's objective function and the participation constraint, there is a direct link between $\mu_{b,t}(y^t)$ and $\gamma(y^t)$.

More precisely, the borrower's Pareto weight evolves according to $\mu_{b,t+1}(y^t) = \mu_{b,t}(y^t) + \gamma(y^t)$, while the lenders' Pareto weight, $\mu_{l,t+1}(y^t)$, remains constant.

Following Marcet and Marimon (2019), the saddle-point Lagrangian problem is homogenous of degree one in $(\mu_{b,t}(y^t), \mu_{l,t}(y^t))$. I can therefore redefine the maximization problem over $(x_t(y^t), 1)$ where $x_t(y^t) = \frac{\mu_{b,t}(y^t)}{\mu_{l,t}(y^t)}$ corresponds to the relative Pareto weight – i.e. the Pareto weight attributed to the borrower relative to the lenders. Given that $(\mu_{b,0}, \mu_{l,0}) > 0$ and $\gamma(y^t) \geq 0$ for all t, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} < \infty$. Moreover,

$$x_{t+1}(y^t) = (1 + \nu(y^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}}, \tag{13}$$

where $\eta \equiv \beta(1+r) < 1$ corresponds to the borrower's impatience relative to the lenders and $\nu(y^t) \equiv \frac{\gamma(y^t)}{\mu_{b,t}(y^t)}$ represents the normalized multiplier attached to the participation constraint. The state vector is simply (y, x) and the Saddle-Point Functional Equation reads

$$\begin{aligned} PV(y,x) = \mathcal{SP} \min_{\nu(y)} \max_{c} \; & x\Big[(1+\nu(y))u(c) - \nu(y)V^D(y)\Big] \\ & + y - c + \frac{1}{1+r}\sum_{y'} \pi(y') PV(y', x') \\ \text{s.t.} \quad & x'(y) = (1+\nu(y))\eta x \;\forall y. \end{aligned} \tag{14}$$

The value function takes the form of $PV(y,x) = xV^b(y,x) + V^l(y,x)$ with $V^b(y,x) = u(c) + \beta\mathbb{E}\left[V^b(y',x')\right]$ being the value of the borrower and $V^l(y,x) = y - c + \frac{1}{1+r}\mathbb{E}\left[V^l(y',x')\right]$ being the value of the lenders. I obtain the optimal consumption by taking the first-order conditions in (14)

$$u_c(c) = \frac{1}{x(1+\nu(y))}. \tag{15}$$

The binding participation constraint of the borrower (i.e. $\nu(y) > 0$) induces an increase in consumption. In what follows, I formalize this argument in Proposition 4.

4.2 Constrained efficient allocation

I characterize the main properties of the Planner's allocation in terms of Pareto frontier and risk sharing. I start with the definition of a threshold value for the relative Pareto weight $x_D(y)$ which is such that $V^b(y, x_D(y)) = V^D(y)$. In words, $x_D(y)$ is the weight at which the participation constraint binds in y. Given this, the following proposition highlights the main properties of the constrained efficient allocation.

Proposition 4 (Constrained Efficient Allocation)**.**

I. (Efficiency). *$V^l(y, x)$ is strictly decreasing, while $V^b(y, x)$ is strictly increasing in $x \in \tilde{X} \equiv [x_D(y_L), \overline{x}]$ for all $y \in Y$ and $x_D(y_H) > x_D(y_L)$.*

II. (Risk-Sharing). *$c(y_L, x) < c(y_H, x)$ and $x'(y_L, x) < x'(y_H, x)$ for $x < x_D(y_H)$ and $c(y_L, x) = c(y_H, x)$ and $x'(y_L, x) = x'(y_H, x)$ otherwise. Also, $c(y_L, x_D(y_L)) = y_L$ and $c(y_H, x_D(y_H)) < y_H$.*

III. (Liabilities). *$V^l(y_L, x) < V^l(y_H, x)$ for all $x \in \tilde{X}$.*

Part I states that the allocation is constrained efficient. Accounting for the participation constraint, it is not possible to make one of the contracting parties better off without making the other worse off.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind in all endowment states. Otherwise, the Planner provides more consumption and a greater continuation value in the high endowment state. Moreover, when the participation constraint binds in y_L, the borrower consumes the value of its endowment. In opposition, when the participation constraint binds in y_H, it consumes less than its endowment indicating a capital outflow consistent with a buyback.

Part III relates to the liabilities of the borrower. In this environment, the value of the lenders represents the net foreign asset position in the contract. A positive value of $V^l(y, x)$ indicates the extent towards which the borrower is indebted. The proposition states that the liabilities increase when y is high. This implies that the Planner adopts a state-contingent debt re-valuation. In the market economy, the payment of the buyback premium χ replicates this state contingency as one will see.

The Planner rules out the autarkic allocation whenever there are strictly positive rents to be shared among the contracting parties. That is why I assume the following.

Assumption 1 (Interiority)**.** *For all $y^t, t \geq 0$, there is a sequence $\{\tilde{c}(y^t)\}$ satisfying*

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(\tilde{c}(y^j)) - V^D(y_t) > 0.$$

Assumption 1 not only rules out autarky as a feasible allocation, it also ensures the uniform boundedness of the Lagrange multipliers. This guarantees existence and uniqueness of the Planner allocation.[6]

[6]See also Proposition 4.10 in Alvarez and Jermann (2000).

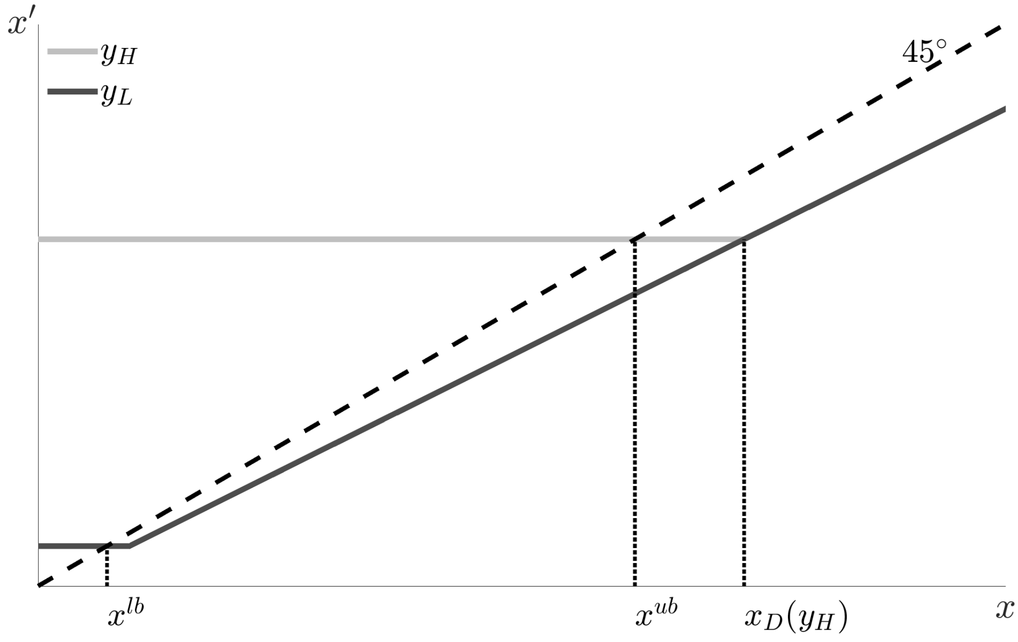
Proposition 5 (Existence and Uniqueness)**.** *Under Assumption 1, given initial conditions* (y_0, x_0)*, there exists a unique constrained efficient allocation.*

Having shown existence and uniqueness of the contract allocation, the following proposition derives the inverse Euler Equation which characterizes the consumption dynamic.

Proposition 6 (Inverse Euler Equation)**.** *The inverse Euler equation is given by*

$$\mathbb{E}\left[\frac{1}{u_c(c(y'))(1+\nu(y'))}\right] = \eta\frac{1}{u_c(c(y))},$$

If the participation constraint never binds, I obtain that $\frac{1}{u_c(c(y))} > \mathbb{E}[\frac{1}{u_c(c(y'))}]$. In this case, the inverse marginal utility of consumption is a positive super-martingale which converges almost surely to 0 by Doob's theorem. Hence, there is a form of immiseration coming from the borrower's relative impatience.[7] With the borrower's limited commitment (i.e. $\nu(y) \geq 0$) and $\eta = 1$, one obtains a left bounded positive sub-martingale. The borrower's participation constraints therefore sets an upper bound on the super-martingale and limits immiseration.



Note: The figure depicts the law of motion of the relative Pareto weight. The light grey line corresponds to the law of motion in y_H and the dark grey line to the law of motion in y_L. The black dotted line represents the 45° line. x^{lb} and x^{ub} correspond to the lower and upper bounds of the ergodic set, respectively. $x_D(y)$ corresponds to the relative Pareto weight at which the participation constraint binds in y.

Figure 1: Steady State Dynamic

[7]The term immiseration is usually used in the context of moral hazard (Thomas and Worrall, 1990; Atkeson and Lucas, 1992).

Given this, I can show that the long-term allocation is characterized by an ergodic set of relative Pareto weights. The term ergodic refers to the fact that the relative Pareto weights in this set are aperiodic and recurrent with non-zero probability.

Proposition 7 (Ergodic Set)**.** *The ergodic set of relative Pareto weights* $[x^{lb}, x^{ub}] \subset \tilde{X}$ *is such that* $x'(y_H, x^{ub}) = x^{ub}$, $x'(y_L, x^{lb}) = x^{lb}$ *with* $x^{lb} = x_D(y_L) < x^{ub} < x_D(y_H)$*.*

The proposition states that the steady state is dynamic and bounded in the interval $[x^{lb}, x^{ub}]$. For instance, after a sufficiently long series of y_L (y_H), the economy hits x^{lb} (x^{ub}). It then stays there until y_H (y_L) realizes and that irrespective of the past realizations of the shock. Figure 1 depicts the law of motion of the relative Pareto weight. The ergodic set corresponds to the interval $[x^{lb}, x^{ub}]$ and the basin of attraction to $(x^{ub}, \overline{x}]$.

4.3 Decentralization

Having derived and characterized the constrained efficient allocation, I construct a Markov equilibrium that decentralizes the Planner's allocation in the market economy. That is, I show the conditions for the Second Welfare Theorem to hold.

Proposition 8 (Decentralization)**.** *Under Assumption 1 and* $x_0 \leq x^{ub}$*, a Markov equilibrium in the market economy decentralizes the constrained efficient allocation.*

Proof. The proof of this proposition is by construction. I first determine the default and buyback policies necessary to implement the constrained efficient allocation. I subsequently derive the underlying portfolio of bonds. Finally, I verify the conditions to enforce the specified default and buyback policies.

Similar to Dovis (2019), I express the policy functions of the implemented Planner's allocation as a function of (y, x). Formally, define $\bar{D}, \bar{M} : Y \times X \to \{0, 1\}$ and $\bar{q}_{st}, \bar{q}_{lt}, \bar{b}_{st}, \bar{b}_{lt} : Y \times X \to \mathbb{R}$. Given the timing of actions, the bond policies and the price schedules can be rewritten as $\bar{b}_j(y, x) = \bar{b}_j(x'(y, x))$ and $\bar{q}_j(y, x) = \bar{q}_j(x'(y, x))$ for all $j \in \{st, lt\}$.

To generate the appropriate state contingency, I assume that buybacks arise when the economy draws y_H and $x < x^{ub}$,

$$\bar{M}(y, x) = \begin{cases} 1 & \text{if} \quad y = y_H \text{ and } x < x^{ub} \\ 0 & \text{else} \end{cases} \tag{16}$$

In addition, given Assumption 1, autarky is never optimal meaning that $\bar{D}(y, x) = 0$ for all

(y, x). Given these two policies and (7)-(8), the bond prices are given by

$$\bar{q}_{st}(x') = \frac{1}{1+r} \quad \text{and} \quad \bar{q}_{lt}(x') = \begin{cases} \frac{1}{1+r}\mathbb{E}[1 + \delta\bar{q}_{lt}(x') + \bar{M}(y', x')\delta\chi] & \text{if } \bar{b}_{lt}(x') < 0 \\ \frac{1}{1+r}\mathbb{E}[1 + \delta\bar{q}_{lt}(x')] & \text{else} \end{cases}$$

Under the buyback policy (16), the long-term bond price is state contingent and increases in the realisation of y_L as shown in the following lemma.

Lemma 2 (Long-Term Bond Price). *Under (16), the long-term bond price is the unique fixed point of $\bar{q}_{lt}$, is decreasing and*

$$\frac{1+\delta\chi}{1+r-\delta} > \bar{q}_{lt}(x'(y_L, x)) \geq \bar{q}_{lt}(x'(y_H, x)) \geq \frac{1}{1+r-\delta},$$

with strict inequality if $\bar{b}_{lt}(x'(y_L, x)) < 0$.

The long-term bond price is higher in y_L than in y_H because a buyback arises in the transition from y_L to y_H. Having properly determined the prices, I can specify the bond portfolio necessary to replicate the constrained efficient allocation. When $x < x^{ub}$,

$$-V^l(y_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_H, x)) + \delta\chi], \tag{17}$$

$$-V^l(y_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))]. \tag{18}$$

In opposition, for $x = x^{ub}$, the relationship is given by

$$-V^l(y_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_H, x))],$$

$$-V^l(y_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma 2 ensures that there exists a unique solution as long as $\bar{b}_{lt}(x'(y_L, x)) < 0$.[8] The bond portfolio is therefore determined for a given χ. With this, I can consider χ as a free parameter.

One needs to check that whenever $y = y_H$ and $x < x^{ub}$, there is effectively a buyback (i.e. $\bar{b}_{lt}(x'(y_H, x)) - \delta\bar{b}_{lt}(x) > 0$). Combining (17) and (18), the short and long-term bond holdings when $x < x^{ub}$ are respectively

$$\bar{b}_{st}(x) = \frac{V^l(y_H, x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))] - V^l(y_L, x)[1 + \delta\bar{q}_{lt}(x'(y_H, x)) + \delta\chi]}{\delta[q_{lt}(x'(y_H, x)) + \chi - \bar{q}_{lt}(x'(y_L, x))]},$$

[8] Observe that there needs to be at least as many maturities as endowment states.

$$\bar{b}_{lt}(x) = -\frac{V^l(y_H, x) - V^l(y_L, x)}{\delta[\bar{q}_{lt}(x'(y_H, x)) + \chi - \bar{q}_{lt}(x'(y_L, x))]} < 0,$$

where $\bar{b}_{lt}(x) < 0$ comes from the fact that $V^l(y_H, x) > V^l(y_L, x)$ given Part III of Proposition 4 and from $\bar{q}_{lt}(x'(y_L, x)) - \bar{q}_{lt}(x'(y_H, x)) = \pi(y_H)\frac{\delta\chi}{1+r} < \chi$. In opposition, at $x \geq x^{ub}$,

$$\bar{b}_{lt}(x) = -\frac{V^l(y_H, x) - V^l(y_L, x)}{\delta[\bar{q}_{lt}(x'(y_H, x)) - \bar{q}_{lt}(x'(y_L, x))]} > 0,$$

From Proposition 7, $x'(y_H, x) = x^{ub}$ for all $x < x^{ub}$. Thus, there is a buyback with $\bar{b}_{lt}(x^{ub}) > 0$ and $\delta\bar{b}_{lt}(x) < 0$. Note further that at $x = x^{ub}$ there is no buyback as $\bar{b}_{lt}(x'(y_H, x^{ub})) = \bar{b}_{lt}(x^{ub})$ from Proposition 7. More generally, at $x \geq x^{ub}$, the borrower possesses a long-term asset so there is no buyback.

Following Alvarez and Jermann (2000), to ensure that the participation constraint (11) holds, it should be that for all $y' \in Y$ and for all $x' \in \tilde{X}$

$$\bar{b}_{st}(x') + \bar{b}_{lt}(x')[1 + \delta\bar{q}_{lt}(x'(y', x')) + \bar{M}(y', x')\delta\chi] \geq \mathcal{G}(y'), \tag{19}$$

where $\mathcal{G}(y') = \bar{b}_{st}(x_D(y')) + \bar{b}_{lt}(x_D(y'))[1 + \bar{M}(y', x_D(y'))\delta\chi + \delta\bar{q}_{lt}(x_D(y'))]$ which by definition of $x_D(y')$ ensures that for all $y' \in Y$ and for all $x' \in \tilde{X}$

$$V^b(y', x') \geq V^D(y').$$

The buyback policy (16) ensures the sate contingency of $\mathcal{G}(y')$. Notice that $V^b(y_H, x) = V^D(y_H)$ for all $x \leq x^{ub}$ as $x^{ub} > x_D(y_H)$ from proposition 7. Thus, (9) ensures the enforcement of $\mathcal{G}(y_H)$. Moreover, I have shown above that $\bar{b}_{lt}(x'(y_L, x)) < 0$ for all $x \leq x^{ub}$. Hence, Proposition 3 Part I ensures that the lenders will not lend more than $\mathcal{G}(y_L)$.

Regarding buybacks, (9) implies that buybacks occur when y_H and $x \leq x^{ub}$ consistent with (16). Moreover, Proposition 3 Part III states that there is no buyback in y_L. Finally, the initial condition $x_0 \leq x^{ub}$ ensures that $x \leq x^{ub}$ following Proposition 7. If $x > x^{ub}$, then $b_{lt}(x) > 0$ implying that $q_{lt}(x'(y_L, x)) = q_{lt}(x'(y_H, x))$.[9] In this case, there is no state contingency in the long-term bond price and it is not possible to decentralize.

This concludes the proof. I used the budget constraints in (2) and in (3) to determine the optimal bond holdings given the prices computed according to (7) and (8). The absence of defaults and the presence of buybacks ensure that (10) is satisfied. Also, buybacks follow (9). Finally, the participation constraint (11) holds. □

[9]Notice that this only holds true under i.i.d endowment shocks. It is therefore possible to relax the assumption that $x_0 \leq x^{ub}$ under persistent endowment shocks.

The implementation works as follows. The borrower conducts buybacks when the economy transits from y_L to y_H. This is when the borrower's participation constraint binds which enables a buyback according to (9). The payment of the buyback premium increases the value of outstanding long-term debt, while the value of short-term bonds remain unchanged. This generates the capital losses and gains necessary to replicate the state contingency of the constrained efficient allocation.[10] In particular, the borrower incurs a capital loss in y_H when paying χ and a capital gain in y_L. The implementation requires that $x_0 < x^{ub}$ meaning that the market economy starts with a sufficiently high level of indebtedness. Otherwise, the borrower would start with long-term assets eliminating the possibility to buyback and with it the state contingency of the long-term bond.

The implementation does not rely on debt restructurings. It is possible to generate the same state contingency with a debt relief in y_L when $x = x_D(y_L)$. As shown by Dovis (2019), this would lead to $\bar{q}_{lt}(x'(y_L, x)) < \bar{q}_{lt}(x'(y_H, x))$ and $b_{lt}(x) < 0$ for all $x \in \tilde{X}$.[11] However, the lenders would never allow this to happen as this involves dilution of legacy long-term debt as shown in Proposition 3 Part I.

Rather than having debt relief in the low endowment state, my implementation builds on debt enlargement in the high endowment state. This gives a clear interpretation to the binding participation constraint in y_H. In this class of model, the borrower's participation constraint binds in the transition from y_L to y_H. This has often been wrongly interpreted as "defaults happen in good times". Under the above mechanism, I show that the borrower's binding constraint in y_H can be interpreted as a buyback. As shown in Proposition 4, the high endowment state is also associated with lower levels of indebtedness and capital outflows, aligning with the idea of a buyback.

The presence of buybacks and the absence of defaults render the long-term bond interest rate spread negative. On the one hand, in the absence of defaults, there is no positive spread. On the other hand, buybacks entail a premium χ implying that the long-term bond price exceeds $\overline{q}$. A negative interest rate spread is a feature that one finds in other decentralizations such as the ones of Liu et al. (2023) and Ábrahám et al. (2025).

The enforcement mechanism in the market economy relies on two pillars. First, the take-it-or-leave-it offer of the incumbent ensures the borrowing limit is satisfied in y_H. Second, the default aversion of the lenders ensure the borrowing limit is satisfied in y_L. In the Online Appendix B, I explore alternatives to buybacks. Empirically, such alternatives do not exist or remain underdeveloped. Moreover, they raise similar enforcement issues as buybacks.

[10] In fact, the notional payment of the two bonds replicates the payout of a short position in Arrow securities, while combined with the buyback premium it replicates the payout of a long position.

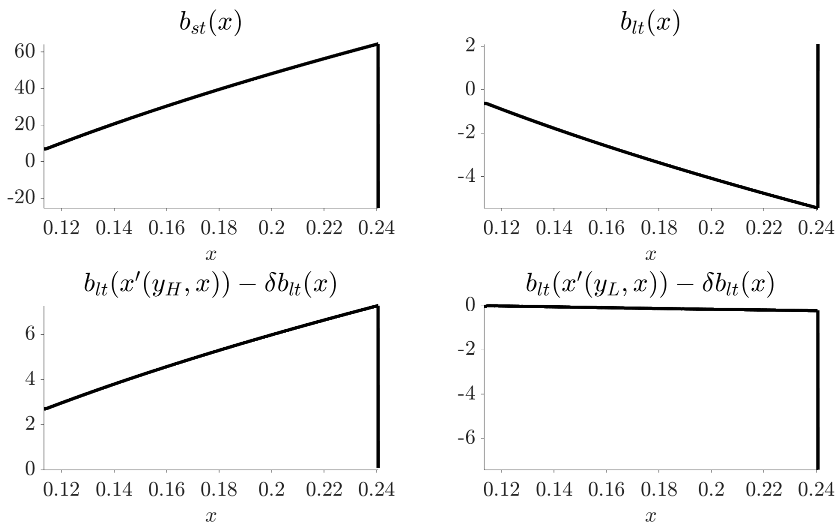
[11] See also Müller et al. (2019) and Restrepo-Echavarria (2019) who interpret the borrower's binding constraint in y_L as a form of preemptive restructuring.

5 Quantitative Analysis

This section presents the quantitative results on buybacks. I first solve the model numerically and present the main policy functions. I then analyze buybacks in Brazil and how they relate to the model's predictions. The details on the data used in this section can be found in the Online Appendix C.

5.1 Numerical solution

I provide a numerical example of the model. The utility function takes the CRRA form $u(c) = \frac{c^{1-\varpi}}{1-\varpi}$. I consider the following parameters: $\varpi = 2$, $\beta = 0.956$, $r = 0.04$, $\pi(y_H) = 0.7892$, $y_L = 0.337$, $y_H = 0.554$, $\delta = 0.95$, $\chi = 0.1494$. The parameters related to the endowment process come from a 2-state Markov chain estimation from Brazil's real GDP between 1999q4 to 2019q4. The buyback premium comes from the estimation presented at the end of this section. The remaining parameters reflect standard values in the literature.



Note: The figure depicts the bond policy functions, which decentralize the constrained efficient allocation, as a function of the relative Pareto weight in the interval $[x^{lb}, x^{ub}]$.

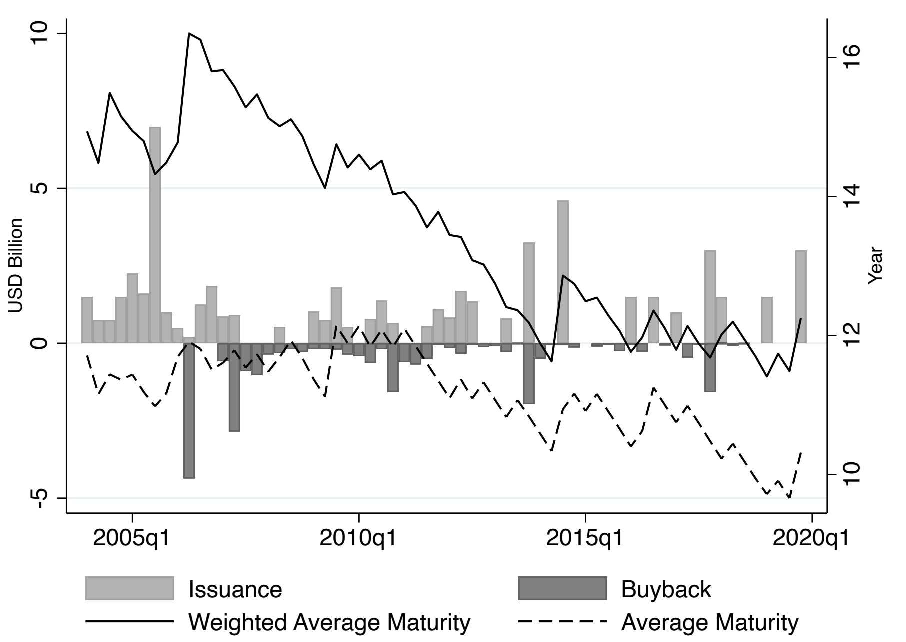
Figure 2: Bond Policy Functions

Figure 2 depicts the bond policy functions that implement the constrained efficient allocation in the interval $[x^{lb}, x^{ub}]$. As one can see the borrower holds short-term assets before the buyback and then switches to debt. The opposite holds true for long-term bonds. Moreover, a buyback arises everytime y_H realizes and $x < x^{ub}$ (i.e. $b_{lt}(x'(y_H, x)) - \delta b_{lt} > 0$). When y_L realizes, there is a net long-term debt issuance (i.e. $b_{lt}(x'(y_L, x)) - \delta b_{lt} < 0$). Finally, the total indebtedness increases as x reduces. Given Proposition 7 this means that buybacks lead to a reduction in indebtedness.

Consistent with the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), the borrower holds long-term and short-term bonds in the magnitude of several multiples of GDP. Even though I consider an alternative environment without commitment, the bond portfolio implementing the constrained efficient allocation still implies large movements in bond holdings.

5.2 Debt issuance and buyback

Brazil defaulted last in the 1980s and regained access to the international market after the implementation of the Brady Plan in 1994.[12] In 2006, the Brazilian government started the Early Redemption Program which aimed at correcting the average maturity of the foreign-currency debt and reducing the potential refinancing risk. Repurchases were conducted by the Brazilian National Treasury either directly on the secondary market or indirectly through tender offers.[13] All figures below relate to USD-denominated bonds.



Note: The figure depicts the issuance, buybacks, weighted average maturity and average maturity of USD-denominated Brazilian bonds from 2004q1 to 2019q4. Weights for the maturity corresponds to the amount of bond outstanding over the total of USD-denominated bonds.

Figure 3: Issuance, Buyback and Average Maturity

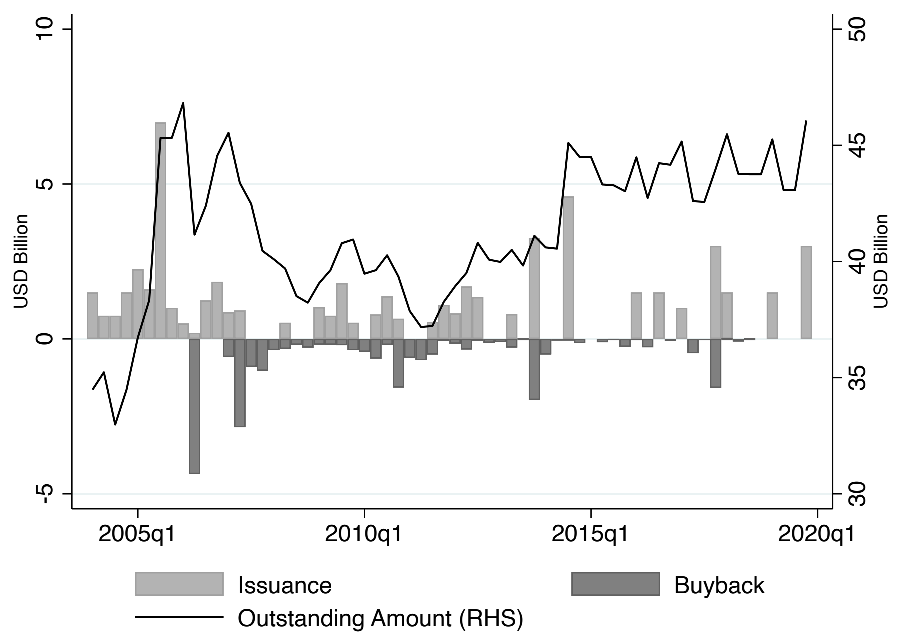
Figure 3 depicts the average maturity of USD-denominated Brazilian bonds alongside issuances and buybacks for the period 2004q1 to 2019q4. Starting in 2006 buybacks led to a

[12] The Brady Plan is an extensive debt restructuring program aimed at resolving the numerous of sovereign debt defaults in 1980s especially in Latin America.

[13] See Ayres et al. (2021) for the economic history of Brazil.

substantial decrease in the weighted average maturity. Following large new issuances at the end of 2014, the weighted average maturity then stabilized at 12 years.

Figure 4 depicts the outstanding amount of USD-denominated Brazilian bonds alongside issuances and buybacks for the period 2004q1 to 2019q4. Similar to maturity, starting in 2006, buybacks led to a decrease in indebtedness. However, the large new issuances at the end of 2014 brought back indebtedness at the level it was in 2005.



Note: The figure depicts the issuance, buybacks, outstanding amount of USD-denominated Brazilian bonds from 2004q1 to 2019q4. Outstanding amounts correspond to the axis on the right-hand side.

Figure 4: Issuance, Buyback and Outstanding Amount

Table 1 depicts the outcome of OLS regressions of respectively buybacks and net issuances in USD billion on the real GDP growth and the EMBI+ spread for the period 2004q1 to 2019q4. Net issuances correspond to issuances minus buybacks. As one can see buybacks arise in periods of high real GDP growth and low EMBI+ spread. Such correlations are economically and statistically significant. If one looks at the net issuance, the opposite relationship holds but is not statistically significant. This is because issuances occur regularly in parallel to buybacks.

As a result, it seems that the evidence from the Brazilian buyback program aligns with the model prediction. Buybacks happen when output grows and the default risk is low. Moreover, they result in a substantial decrease in indebtedness and in the weighted average maturity. This is especially true at the beginning of the buyback program where the largest repurchases occurred.

Table 1: Buyback and Issuance Regression

	(1) Buyback	(2) Buyback	(3) Net Issuance	(4) Net Issuance
Real GDP Growth	0.05***		-0.05	
	[0.02]		[0.04]	
EMBI+ Spread		-0.18***		0.31**
		[0.06]		[0.15]
Observations	64	64	64	64
R^2	0.05	0.07	0.01	0.05

Note: The table depicts the outcome of OLS regressions of respectively buybacks and net issuances in USD billion on the real GDP growth and the EMBI+ spread for the period 2004q1 to 2019q4. Net issuances correspond to issuances minus buybacks. Robust standard errors are in squared brackets. A coefficient with *** has a statistical significance of 1%, ** of 5% and * of 10%.

5.3 Buyback premium

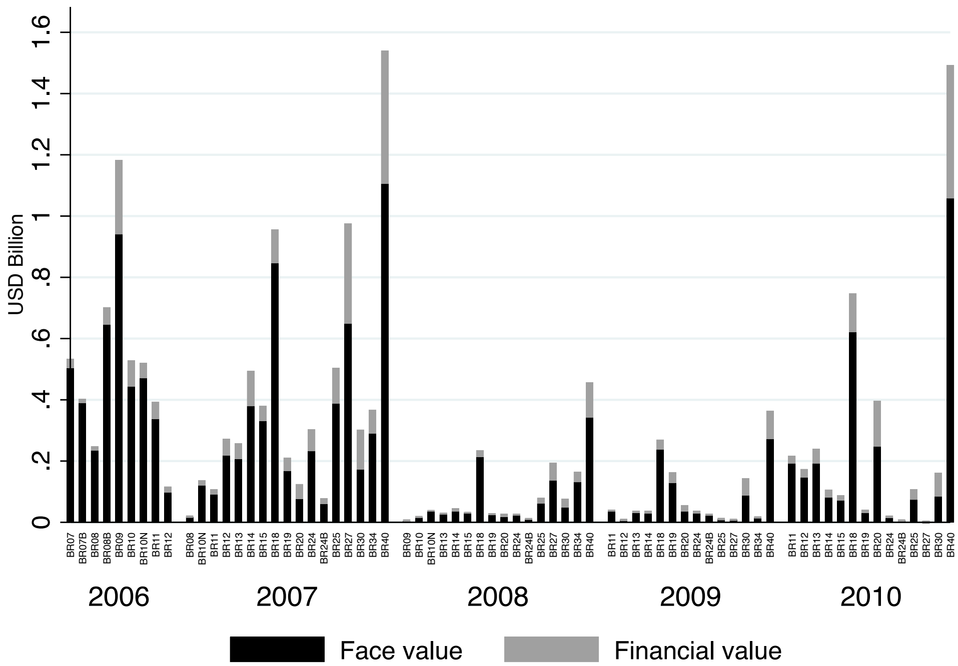
One of the main elements in the theoretical analysis is that the borrower pays a premium χ on the secondary market. I now test whether one observes such a premium in the data.

Figure 5 depicts the amount bought back by the Brazilian government between 2006 and 2018 for bonds denominated in USD and issued prior to 2006.[14] The black bar represents the face value, while the grey bar corresponds to the financial value. In the period considered, buybacks amounted to a total of 21 USD billion in face value and 27 USD billion in financial value.[15] Financial value buybacks were the largest in 2007 with 7 USD billion, in 2006 with 5 USD billion and in 2010 with 4 USD billion. The same holds true for face value buybacks with a total amount of 5 USD billion in 2007, 4 USD billion in 2006 and 3 USD billion in 2010. In addition, buybacks were the largest for bonds with a relatively long residual maturity. For instance, buybacks for bonds due in 10 years or more amounted to 10 USD billions in face value and 14 USD billions in financial value.

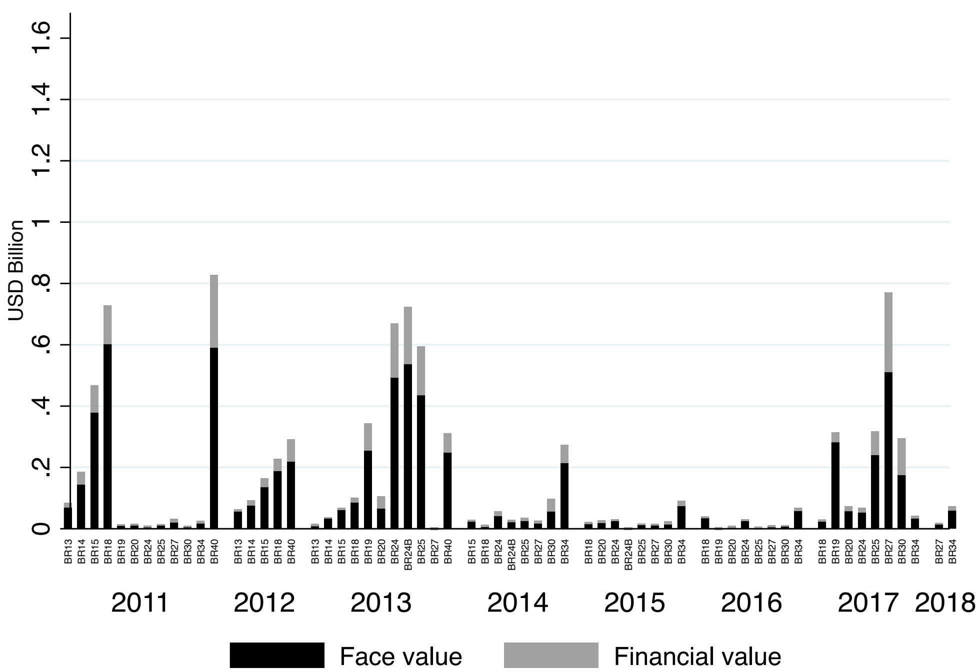
In the model, the buyback premium reflects the capital gain of the incumbent lender. A bond contract specifies a sequence of coupon payments and a principal repayment at maturity. A borrower issues bonds on the primary market at a given price. In the absence of a buyback, the borrower receives the value of the bond at the primary-market price and transfers to the lenders the coupons and the principal when due up to maturity. In the presence of a buyback, the borrower transfers the coupons due up to the buyback and the secondary-market price of the bond during the buyback. I therefore estimate the buyback premium by comparing the cashflow stream of a bond with and without a buyback.

[14] I also omit a bond with floating coupon (BR09F) due to the lack of information on the coupon structure. Note further that there is no buyback in 2019.

[15] Buybacks for bonds issued from 2006 onwards amounts to an additional 2 USD billion in both face and financial values.



(a) 2006-2010



(b) 2011-2018

Note: The figure depicts the buyback amount by year and by bonds. All bonds are denominated in USD and issued prior to 2006. The Financial value corresponds to the amount required for payment of the securities redeemed, while the face value corresponds to the value of debt in the national statistics. There is no buyback in 2019.

Figure 5: Buyback Amount by Bond and Year

Formally, consider a bond with a face value b, a coupon rate κ, a yield to maturity i and a maturity T. The present value of the coupons is given by $FVC(T, \kappa, b; i) = \sum_{t=1}^{T} \frac{b\kappa}{(1+i)^t} = \frac{1-(1+i)^{-T}}{i} b\kappa$. The present value of the face value corresponds to $FVP(T, b; i) = \frac{b}{(1+i)^T}$. Given this, in the absence of a buyback, the financial value of a bond for a given i is

$$FV^{NB}(T, \kappa, b; i) = FVC(T, \kappa, b; i) + FVP(T, b; i).$$

Evaluated at the issue yield to maturity, this equation gives the value the lenders get if they hold the bond from the issuance to the maturity.

In the case of a buyback, the borrower repurchases the bond prior to maturity, say $t_B < T$. The bond is repurchased on the secondary market at a financial value $B = FVC(T - t_B, \kappa, b; i_B) + FVP(T - t_B, b; i_B)$ where i_B is the yield to maturity at t_B. Thus, in the presence of buybacks, the financial value of a bond for a given i is

$$FV^B(t_B, \kappa, b, B; i) = FVC(t_B, \kappa, b; i) + FVP(t_B, B; i).$$

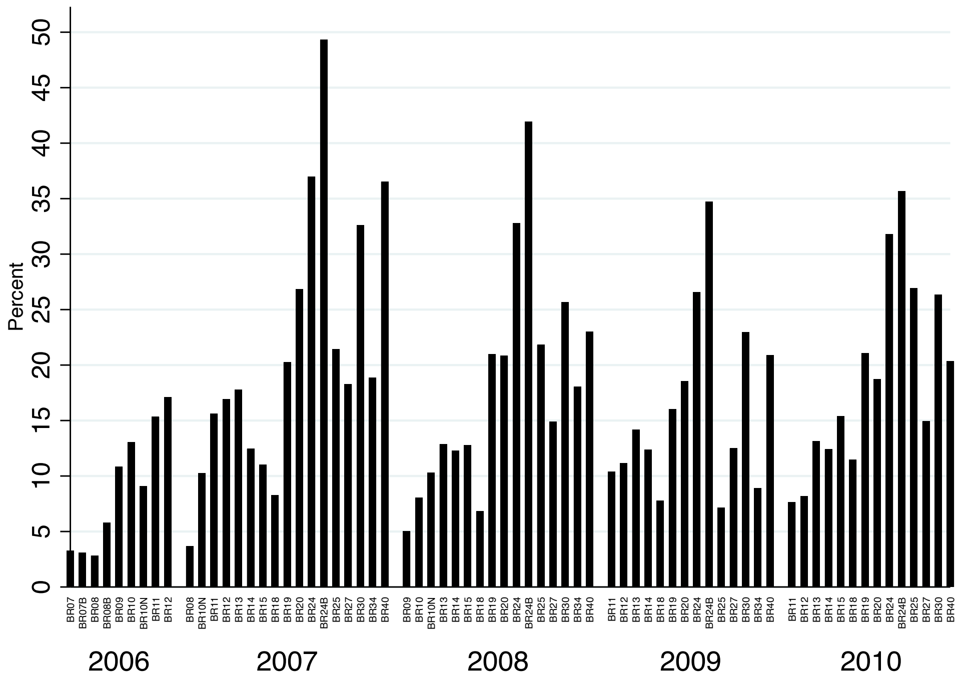
The yield to maturity represents the internal rate of return of the bond. Hence, whenever $FV^B(t_B, \kappa, b, B; i) > FV^{NB}(T, \kappa, b; i)$ for a given i, the borrower pays a premium in the buyback operation. In the opposite case, there is a discount.

As the borrower issues bonds on the primary market and potentially buys them back on the secondary market, there are two yields to maturity to consider. The first one is the issue yield to maturity, i_I, which gives the bond's expected total return at issuance. The second one is i_B necessary to compute B. The buyback premium at issuance is therefore

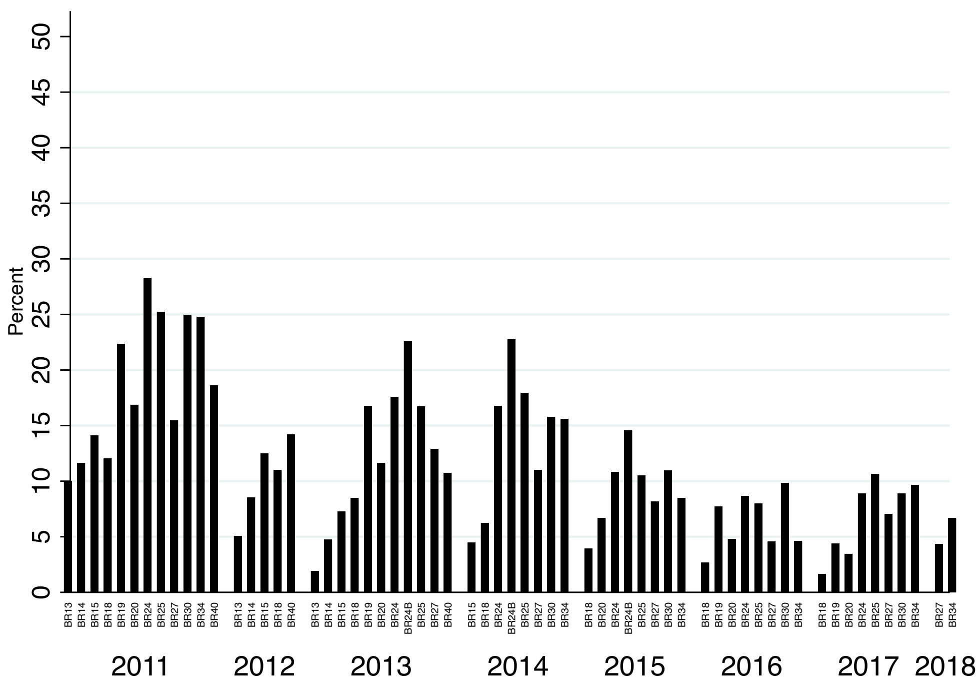
$$\frac{FV^B(t_B, \kappa, b, B, i_I)}{FV^{NB}(T, \kappa, b, i_I)} - 1. \tag{20}$$

Note that I restrict my attention to bonds issued prior to the start of the Early Redemption Program. This limits the extent towards which the yield to maturity at issuance anticipates futures buybacks. In the case of re-openings, I take the minimum yield to maturity between issuance and the different re-openings. Taking a weighted average instead does not significantly affect my results.

The computation of the buyback premium follows a similar logic as the computation of haircuts in Sturzenegger and Zettelmeyer (2008). Equation (20) compares the present value of the "old" debt (denominator) with the present value of the "new" debt (numerator). In that logic the premium corresponds to a negative haircut. I do not compute a "market" premium which would consist of replacing $FV^{NB}(T, \kappa, b, i_I)$ by its face value in (20). The reason is that this measure would exaggerate the premium paid by the borrower.



(a) 2006-2010



(b) 2011-2018

Note: The figure depicts the average buyback premium by year and by bond. The buyback premium is computed according to (20).

Figure 6: Buyback Premium by Bond and Year

Figure 6 depicts the average buyback premium in percent by year and by bond. The premium averages 14.94% and has a median at 13.07% overall. The maximum and the minimum premium amounts to 53.59% and 0.01%, respectively. Three points deserved to be noted. First, the buyback premium is always positive. This comes from the fact that bonds were usually issued below the par value and bought back above the par value. As one can see in Table 2, the issue yield to maturity is almost always larger than the coupon rate. However, the opposite is true for the buyback yield to maturity. Second, the Brazilian government bought back on average 48.73% of the total issuance. Third, the bonds involved in the buyback have a (weighted) average maturity of 16 (19) years at issuance.

Table 2: Buyback Amount, Yield and Premium by Bond

Bond	Maturity (Year)	Coupon Rate (Percent)	Total Issuance (USD Billion)	Average i_I (Percent)	Total Buyback (USD Billion)	Average i_B (Percent)	Average Premium (Percent)
BR07	7	11.25	1.50	11.13	0.50	6.02	3.24
BR07B	4	10.00	1.00	10.70	0.39	4.99	3.07
BR08	10	9.38	1.25	9.40	0.25	3.44	3.25
BR08B	6	11.50	1.25	11.74	0.65	6.02	5.80
BR09	10	14.50	2.00	14.61	0.95	4.23	7.92
BR10	8	12.00	1.00	12.38	0.46	3.65	10.54
BR10N	7	9.25	1.50	9.45	0.63	4.31	10.11
BR11	9	10.00	1.25	10.66	0.65	5.09	13.69
BR12	10	11.00	1.25	12.60	0.47	3.50	13.39
BR13	10	10.25	1.25	10.58	0.59	0.11	10.75
BR14	10	10.50	1.25	8.24	0.78	2.25	10.41
BR15	10	7.88	2.10	7.73	1.03	2.20	10.96
BR18	13	8.00	4.51	7.58	2.89	4.54	7.87
BR19	15	8.88	1.50	8.83	0.90	4.44	15.84
BR20	20	12.75	1.00	13.27	0.53	4.30	15.51
BR24	23	8.88	2.15	12.91	0.94	5.27	22.05
BR24B	21	8.88	0.82	12.59	0.66	5.54	34.30
BR25	20	8.75	2.25	8.52	1.26	5.28	17.05
BR27	30	10.13	3.50	10.29	1.38	5.60	12.37
BR30	30	12.25	1.60	12.47	0.65	5.63	21.49
BR34	30	8.25	2.70	8.24	0.89	6.15	13.09
BR40	40	11.00	5.16	13.73	3.84	7.93	20.45

Note: The table depicts the main statistics of bonds bought back by the Brazilian government between 2006 and 2018. All bonds are denominated in USD and issued prior to 2006. For each bond the table gives the maturity at issuance, the coupon rate, the total amount issued (including re-openings), the issue yield to maturity, the total amount bought back, the buyback yield to maturity and the premium computed according to (20).

My estimation strategy differs from the one of Bulow and Rogoff (1988, 1991). The two authors take the difference in the value of debt in the secondary market before and after the buyback. They then compare this difference with the amount paid for the buyback. This comparison reflects a more marked-to-market approach. However, it does not properly capture the capital transfer between the borrower and the lenders in the model as it computes the premium as if the debt was issued in the secondary market prior to the buyback.

6 Conclusion

This paper shows that buybacks can be rationalized as part of an optimal contract between a sovereign borrower and foreign lenders. The bottom line is that buybacks allow bonds to function like Arrow securities. This is because issuances and buybacks occur in two distinct markets. On the one hand, buybacks take place in the secondary market, where the borrower can only deal with legacy lenders. On the other hand, new issuances occur in the primary market, where the borrower deals with legacy lenders and new entrants. Assuming that legacy lenders coordinate, the borrower faces a monopolist in the secondary market and competitors in the primary market leading to a higher payout of legacy long-term debt during buybacks than during issuances.

Furthermore, buybacks provide a clear interpretation for the binding participation constraint in optimal contracts. The property that the borrower's participation constraint binds when endowment increases has often been wrongly interpreted as "defaults happen in good times". Instead, buybacks align with the fact that indebtedness decreases and capital flows out in the high endowment state.

This study stresses the fact that the strategic interaction of the lenders is key. The literature on sovereign debt and default has focused on the borrower's side. However, it is possible to explain a variety of alternative dynamics in equilibrium by looking at the lenders and the way they interact.

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Online Appendix

(Not for Publication)

A Proofs

A.1 Proof of Proposition 1

Proposition 1 (Optimal Offers). *In the primary and the secondary markets, the two lenders offer bond contracts with prices (q_{st}, q_{lt}) satisfying (7)-(8) and quantities (b'_{st}, b'_{lt}) such that*

$$q_{lt}(b'_{st}, b'_{lt}) = \begin{cases} \arg\max_{\tilde{b}'_{st}, \tilde{b}'_{lt} \leq \delta b_{lt}} \left\{ q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt}) \right\} & \text{if } b_{lt} < 0 \text{ and } b'_{lt} \leq \delta b_{lt} \\ \arg\max_{\tilde{b}'_{st}, \tilde{b}'_{lt} > \delta b_{lt}} \left\{ q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt}) \right\} & \text{if } b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt} \end{cases} \tag{10}$$

implying that (5)-(6) hold with equality in all Ω.

Proof. I first prove that the two lenders always offer competitive prices. For this, consider that the two lenders offer the same quantity $(b^{o\prime}_{st}, b^{o\prime}_{lt}) = (b^{i\prime}_{st}, b^{i\prime}_{lt}) = (b'_{st}, b'_{lt})$ but at potentially different prices. Denote by (q_{st}, q_{lt}) the competitive price vector satisfying (7) -(8). I start with the case without buyback meaning that $b'_{lt} < \delta b_{lt}$ if $b_{lt} < 0$ and distinguish 3 cases:

1. Suppose the incumbent offers $(q^i_{st}, q^i_{st}) = (q_{st}, q_{lt})$. If the incumbent's offer is accepted, its payoff is given by

   $$-b_{st} - b_{lt} + q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r} \mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

   Observe that if $b'_{lt} - \delta b_{lt} < 0$, the borrower would be strictly better off with a price higher than q_{lt}. In opposition, if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$, the borrower would be strictly better off with a price lower than q_{lt}. Similarly if $b_{st} < 0$ ($b_{st} > 0$) the borrower would be strictly better off with a price higher (lower) than q_{st}. Hence, depending on the sign of $b'_{lt} - \delta b_{lt}$ and b_{lt}, the outsider can undercut the incumbent offer by appropriately offering a higher or a lower price than the competitive price.

   However, if the outsider offers such prices, it incurs a loss. When $b'_{lt} - \delta b_{lt} < 0$ or $b_{st} < 0$, the outsider is selling debt at a price which is lower than the present value of expected returns. Similarly, when $b'_{lt} - \delta b_{lt} > 0$ or $b_{st} > 0$, it is taking debt at a price which is higher than the present value of expected payouts. If the outsider's offer is accepted, its payoff is

   $$q^o_{st} b'_{st} + q^o_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r} \mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

In opposition, if the outsider offers (q_{st}, q_{lt}) and its offer is accepted, its payoff is

$$q_{st}b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

Note that there is no buyback meaning that $b'_{lt} - \delta b_{lt} < 0$ if $b_{lt} < 0$. As $(q_{st} - q^o_{st})b'_{st} + (q_{lt} - q^o_{lt})(b'_{lt} - \delta b_{lt}) > 0$, the outsider is strictly worse than offering (q_{st}, q_{lt}).

2. Suppose the incumbent offers $q^i_{st} < q_{st}$ if $b'_{st} < 0$, $q^i_{st} > q_{st}$ if $b'_{st} > 0$, $q^i_{lt} < q_{lt}$ if $b'_{lt} - \delta b_{lt} < 0$ and $q^i_{lt} > q_{lt}$ if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$. The outsider can undercut the incumbent's offer by offering $(q^o_{st}, q^o_{st}) = (q_{st}, q_{st})$. Note that (5) holds since $q^m_{lt} = q_{lt}$ in all cases. The borrower strictly prefers the outsider's offer, meaning that the incumbent's payoff is

$$-b_{st} - b_{lt}(1 + \delta q^m_{lt}),$$

which by (5) is weakly lower than the payoff when the incumbent offers (q_{st}, q_{lt}).

3. Suppose the incumbent offers $q^i_{st} > q_{st}$ if $b'_{st} < 0$, $q^i_{st} < q_{st}$ if $b'_{st} > 0$, $q^i_{lt} > q_{lt}$ if $b'_{lt} - \delta b_{lt} < 0$ and $q^i_{lt} < q_{lt}$ if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$. If the outsider offers $(q^o_{st}, q^o_{st}) = (q_{st}, q_{lt})$, the borrower strictly prefers the incumbent's offer. Repeating the argument of the first case, the incumbent is strictly worse than offering (q_{st}, q_{lt}).

Consequently, the two lenders offer competitive prices. For $b_{st} < 0$ and $b'_{lt} - \delta b_{lt} < 0$ ($b_{st} > 0$ and $b'_{lt} - \delta b_{lt} > 0$), they have no incentive to offer a price higher (lower) than (q_{st}, q_{lt}) as this would result to losses. There is also no reason to offer a lower (higher) price than (q_{st}, q_{lt}) as one lender can undercut the other.

In the case with buyback, the two lenders compete on the short-term bond price. It is straightforward to repeat the previous argument in that case. On the secondary market, suppose the incumbent offer a non-competitive price $\ddot{q}_{lt}$. As the borrower picks the entire offer $(b'_{st}, q_{st}; b'_{lt}, \ddot{q}_{lt}, \chi)$, the outsider can adapt its offer on primary market. In particular, it can make an offer entailing a loss on the short-term bond which exactly offset the gain on the long-term bond. The borrower would strictly prefer this offer. The outsider then becomes the incumbent and (6) is satisfied with equality.

I fixed the quantity and changed the price. However, one could have done the opposite. As a result, irrespective on whether the borrower enters the secondary market or not, it holds that

$$q_{st}b'_{st} + q_{lt}b'_{lt} + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})] = 0. \tag{A.1}$$

The equality comes from the fact that competitive prices imply zero expected returns on new bond contracts.

I now turn to the second part of the proposition, namely that any offer satisfies (10). Suppose that $b_{lt} < 0$ and consider two different offers without buyback $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ and $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ satisfying (7) -(8) with $q_{lt} > \tilde{q}_{lt}$. If the offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ is accepted, the incumbent value is

$$-b_{st} - b_{lt}(1 + \delta q_{lt}),$$

where I used (A.1). Conversely, if the offer $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ is accepted,

$$-b_{st} - b_{lt}(1 + \delta \tilde{q}_{lt}).$$

Since $q_{lt} > \tilde{q}_{lt}$, the incumbent would prefer to offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ instead of $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ as $b_{lt} < 0$. Moreover, the outsider is unwilling to offer $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ as this would violate (5) when the incumbent offers $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$. To see this, consider that the outsider offers $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ and it is accepted, its value is given by

$$\delta b_{lt}(q_{lt} - \tilde{q}_{lt}) < 0.$$

The outsider would make losses as it sells a contract at price $\tilde{q}_{lt}$ but must repay the incumbent at price $q_{lt} > \tilde{q}_{lt}$. If $q_{lt} = \max_{\tilde{b}'_{st}, \tilde{b}'_{lt} \leq \delta b_{lt}} \{q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt})\}$ for $b'_{lt} \leq \delta b_{lt}$, the proof is done. Otherwise, we can repeat the previous argument with an offer $(\ddot{b}'_{st}, \ddot{q}_{st}; \ddot{b}'_{lt}, \ddot{q}_{lt})$ such that $\ddot{q}_{lt} > q_{lt}$ until $\ddot{q}_{lt} = \max_{\tilde{b}'_{st}, \tilde{b}'_{lt} \leq \delta b_{lt}} \{q_{lt}(\tilde{b}'_{st}, \tilde{b}'_{lt})\}$. The extension to the case of a buyback with (6) is straightforward. As a result, (A.1) together with (10) imply that (5) and (6) hold with equality. □

A.2 Proof of Proposition 2

Proposition 2 (Optimal Bond Contracts). *In equilibrium, if $M(\Omega) = 0$, the optimal bond contracts $B_{st}(\Omega)$ and $B_{lt}(\Omega)$ solve*

$$V^P_{NB}(\Omega) = \max_{b'_{st}, b'_{lt}} \Big\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \Big\}$$

$$s.t. \quad c + q_{st}(b'_{st}, b'_{lt})b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt},$$

$$b'_{lt} \leq \delta b_{lt} \ \textit{if}\ b_{lt} < 0.$$

(7), (8) *and* (10),

and, if $M(\Omega) = 1$, *the optimal bond contracts* $B_{st}(\Omega)$ *and* $B_{lt}(\Omega)$ *solve for a given* χ

$$
\begin{aligned}
V_B^P(\Omega) = \max_{b'_{st}, b'_{lt}} & \Big\{ u(c) + \beta \mathbb{E}\left[V(\Omega')\right] \Big\} \\
\text{s.t.} \quad & c + q_{st}(b'_{st}, b'_{lt}) b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1 + \delta\chi), \\
& b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt}, \\
& (7), (8) \text{ and } (10),
\end{aligned}
$$

and $\chi(\Omega)$ *satisfies* (9).

Proof. Proposition 1 shows that lenders always offer bond contracts satisfying (7), (8) and (10). I additionally need to show that the lenders' offer needs to maximize the borrower's utility. First consider the case without buyback. Assume by contradiction that the incumbent offers $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that (7), (8) and (10) hold but does not satisfy (2). Then the outsider can make a counter offer such that (7), (8), (10) and (2) hold. The borrower would prefer this offer and the outsider becomes the incumbent. In the case of a buyback, the outsider can only compete with the incumbent on the short-term bond contract. Hence, for the short-term bond contract, the same argument as before holds. For the long-term bond contract, the buyback happens only if the borrower decides to enter the secondary market for a given χ. As a result, the incumbent's offer satisfies (3) and the optimal buyback premium satisfies (9). □

A.3 Proof of Proposition 3

Before proving Proposition 3, I present some intermediate results. First, I show the monotonicity of the value under buyback, no buyback and default.

Proposition A.1. $V_{NB}^P(y, b_{st}, b_{lt})$ *and* $V_B^P(y, b_{st}, b_{lt})$ *are strictly increasing in* (y, b_{st}, b_{lt}) *and* $V^D(y)$ *is strictly increasing in* y.

Proof. Consider that the optimal offer under y_H is $(b_{st}^{H\prime}, q_{st}^H; b_{lt}^{H\prime}, q_{lt}^H)$ and under y_L is $(b_{st}^{L\prime}, q_{st}^L; b_{lt}^{L\prime}, q_{lt}^L)$. One the has that

$$
\begin{aligned}
V_{NB}^P(y_H, b_{st}, b_{lt}) &= u(y_H + b_{st} + b_{lt} - q_{st}^H b_{st}^{H\prime} - q_{lt}^H(b_{lt}^{H\prime} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b_{lt}^{H\prime}, b_{st}^{H\prime})] \\
&\geq u(y_H + b_{st} + b_{lt} - q_{st}^L b_{st}^{L\prime} - q_{lt}^L(b_{lt}^{L\prime} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b_{lt}^{L\prime}, b_{st}^{L\prime})] \\
&> u(y_L + b_{st} + b_{lt} - q_{st}^L b_{st}^{L\prime} - q_{lt}^L(b_{lt}^{L\prime} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b_{lt}^{L\prime}, b_{st}^{L\prime})] \\
&= V_{NB}^P(y_L, b_{st}, b_{lt}),
\end{aligned}
$$

where the first inequality comes from optimality and the second from $y_H > y_L$. The exact same argument can be repeated for $V_B^P(y, b_{st}, b_{lt})$. Moreover,

$$V^D(y_H) = u(y_H) + \beta\mathbb{E}[V^D(y')] > u(y_L) + \beta\mathbb{E}[V^D(y')] = V^D(y_L),$$

where the inequality comes from $y_H > y_L$.

Consider $(b_{st}^1, b_{lt}^1) < (b_{st}^2, b_{lt}^2)$. Denote the optimal offer under (b_{st}^2, b_{lt}^2) by $(b_{st}^{2\prime}, q_{st}^2; b_{lt}^{2\prime}, q_{lt}^2)$ and under (b_{st}^1, b_{lt}^1) by $(b_{st}^{1\prime}, q_{st}^1; b_{lt}^{1\prime}, q_{lt}^1)$. One the has that

$$\begin{aligned} V_{NB}^P(y, b_{st}^2, b_{lt}^2) &= u(y + b_{st}^2 + b_{lt}^2 - q_{st}^2 b_{st}^{2\prime} - q_{lt}^2(b_{lt}^{2\prime} - \delta b_{lt}^2)) + \beta\mathbb{E}[V(y', b_{lt}^{2\prime}, b_{st}^{2\prime})] \\ &\geq u(y + b_{st}^2 + b_{lt}^2 - q_{st}^1 b_{st}^{1\prime} - q_{lt}^1(b_{lt}^{1\prime} - \delta b_{lt}^2)) + \beta\mathbb{E}[V(y', b_{lt}^{1\prime}, b_{st}^{1\prime})] \\ &> u(y + b_{st}^1 + b_{lt}^1 - q_{st}^1 b_{st}^{1\prime} - q_{lt}^1(b_{lt}^{1\prime} - \delta b_{lt}^1)) + \beta\mathbb{E}[V(y', b_{lt}^{1\prime}, b_{st}^{1\prime})] \\ &= V_{NB}^P(y, b_{st}^1, b_{lt}^1), \end{aligned} \tag{A.2}$$

where the first inequality comes from optimality and the second from $(b_{st}^1, b_{lt}^1) < (b_{st}^2, b_{lt}^2)$. The exact same argument can be repeated for $V_B^P(y, b_{st}, b_{lt})$. □

Second, I show that there is no contract with capital inflow under default risk. This is a repetition of Proposition 2 in Arellano (2008).

Proposition A.2. *If for some (b_{st}, b_{lt}), $\mathbb{E}[D(y, b_{st}, b_{lt})] > 0$, then there are no bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $b_{st} + b_{lt} - q_{st}b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) > 0$.*

Proof. The proof follows Arellano (2008, Proposition 2). Suppose by contradiction that there is a contract $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) = b_{st} + b_{lt} - q_{st}b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) > 0$ but the borrower prefers an alternative contract $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ such that $\Delta(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt}) = b_{st} + b_{lt} - \tilde{q}_{st}\tilde{b}'_{st} - \tilde{q}_{lt}(\tilde{b}'_{lt} - \delta b_{lt}) \leq 0$ and finds default to be optimal because $u(y + \Delta(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})) + \beta\mathbb{E}[V(y', \tilde{b}'_{lt}, \tilde{b}'_{lt})] < u(y) + \beta\mathbb{E}[V^D(y')]$.

Observe that under all contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) > 0$, the borrower prefers to repay than to default. This is because $u(y + \Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt})) > u(y)$ and $\mathbb{E}[V(y', b'_{lt}, b'_{lt})] \geq \mathbb{E}[V^D(y')]$ for $b'_{st} + b'_{lt}(1 - \delta q'_{lt}) \leq 0$. This contradicts the fact that both $(\tilde{b}'_{st}, \tilde{q}'_{st}; \tilde{b}'_{lt}, \tilde{q}'_{lt})$ and default are optimal. □

Given Proposition A.2, I can show that the default risk diminishes when y increases if there is no bond contract with capital inflow.

Proposition A.3. *If all bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ are such that $b_{st} + b_{lt} - q_{st}b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) \leq 0$, then $V_{NB}^P(y_H, b_{st}, b_{lt}) - V^D(y_H) \geq V_{NB}^P(y_L, b_{st}, b_{lt}) - V^D(y_L)$.*

Proof. Suppose that all bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ are such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) = b_{st} + b_{lt} - q_{st}b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) \leq 0$. Now consider that the optimal offer under y_H is $(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})$ and under y_L is $(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})$. From optimality, one has that

$$u(y_H + \Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})) + \beta \mathbb{E}\left[V(y', b^{H\prime}_{st}, b^{H\prime}_{lt})\right] \geq u(y_H + \Delta(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})) + \beta \mathbb{E}\left[V(y', b^{L\prime}_{st}, b^{L\prime}_{lt})\right].$$

So if

$$\begin{aligned} &u(y_H) + \beta \mathbb{E}\left[V^D(y')\right] - \Big[u(y_L) + \beta \mathbb{E}\left[V^D(y')\right]\Big] \leq \\ &u(y_H + \Delta(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})) + \beta \mathbb{E}\left[V(y', b^{L\prime}_{st}, b^{L\prime}_{lt})\right] - \Big[u(y_L + \Delta(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})) + \beta \mathbb{E}\left[V(y', b^{L\prime}_{st}, b^{L\prime}_{lt})\right]\Big]. \end{aligned} \tag{A.3}$$

then one gets that

$$\begin{aligned} &u(y_H) + \beta \mathbb{E}\left[V^D(y')\right] - \Big[u(y_L) + \beta \mathbb{E}\left[V^D(y')\right]\Big] \leq \\ &u(y_H + \Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})) + \beta \mathbb{E}\left[V(y', b^{H\prime}_{st}, b^{H\prime}_{lt})\right] - \Big[u(y_L + \Delta(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})) + \beta \mathbb{E}\left[V(y', b^{L\prime}_{st}, b^{L\prime}_{lt})\right]\Big]. \end{aligned}$$

Simplifying (A.3), one obtains that

$$u(y_H) - u(y_L) \leq u(y_H + \Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})) - u(y_L + \Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})),$$

which holds true given the strict concavity of $u(\cdot)$ and the fact that all bond contracts are such that $\Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt}) \leq 0$. □

Given Propositions 1, A.1, A.2 and A.3, I can prove Proposition 3. I first show that the default risk diminishes when $b_{lt} < 0$ and then show in which part of the state space defaults and buybacks do not arise.

Proposition 3 (Default and Buyback)**.**

I. (Long-Term Debt). *For any* $b_{lt} < 0$, $\mathbb{E}[D(y', B_{st}(\Omega), B_{lt}(\Omega))] \leq \mathbb{E}[D(\Omega)]$.

II. (Default). *For any* Ω, $D(y_H, B_{st}(\Omega), B_{lt}(\Omega)) = 0$.

III. (Buyback). *For any* (b_{st}, b_{lt}), $M(y_L, b_{st}, b_{lt}) = 0$.

Proof. I prove each part of the proposition one by one:

– Part I

Consider two cases. First, when $b_{lt} < 0$, the two lenders make an offer that maximizes the long-term bond price as shown in Proposition 1. A close inspection of (8) tells us that $Q_{lt}(\Omega) \geq R_{lt}(\Omega)$ since $b_{st} + b_{lt}(1 + \delta q_{lt}) \leq 0$. In addition, under (9), buybacks either take place instead of default or the opposite. In other words, enabling borrowing beyond the risk-free borrowing limit decreases the long-term bond price. Hence, the only way to maximize the long-term bond price is to make an offer such that $\mathbb{E}[D(y', b'_{st}, b'_{lt})] \leq \mathbb{E}[D(y', b_{st}, b_{lt})]$.

Second, suppose that $b_{lt} > 0$. In that case, the incumbent is borrowing a long-term bond from the borrower. It is willing to dilute to increase its payoff.

– Part II

Denote the bond policy vector $\boldsymbol{B}(\Omega) = (B_{st}(\Omega), B_{lt}(\Omega))$ and fix Ω. From Propositions A.2 and A.3, if there is a risk of default $V^P_{NB}(y_H, \boldsymbol{B}(\Omega)) - V^D(y_H) \geq V^P_{NB}(y_L, \boldsymbol{B}(\Omega)) - V^D(y_L)$. This means that if $D(y_H, \boldsymbol{B}(\Omega)) = 1$ then $D(y_L, \boldsymbol{B}(\Omega)) = 1$. The opposite is however not true. Hence, defaults arise in y_H only if they arise in y_L too. This implies that when $D(y_H, \boldsymbol{B}(\Omega)) = 1$, $\mathbb{E}[D(y, \boldsymbol{B}(\Omega))] = 1$.

At these odds, bond prices are as follows. First, if $b'_{st} < 0$ and $b'_{lt} < 0$, then $q_{st}(b'_{st}, b'_{lt}) = q_{lt}(b'_{st}, b'_{lt}) = 0$ given (7)-(8). Second, if $b'_{st} \geq 0$ and $b'_{lt} < 0$, then $q_{st}(b'_{st}, b'_{lt}) = \frac{1}{1+r}$ and $q_{lt}(b'_{st}, b'_{lt}) = \frac{1}{1+r}\frac{b'_{st}}{-b'_{lt}}$. This implies that $q_{st}(b'_{st}, b'_{lt})b'_{st} = -q_{lt}(b'_{st}, b'_{lt})b'_{lt}$. Third, if $b'_{st} < 0$ and $b'_{lt} \geq 0$, then $q_{st}(b'_{st}, b'_{lt}) = \frac{1}{1+r}\frac{(1+r)q_{lt}(b'_{st}, b'_{lt})b'_{lt}}{-b_{st}}$ and $q_{lt}(b'_{st}, b'_{lt}) > 0$ implying that $q_{st}(b'_{st}, b'_{lt})b'_{st} = -q_{lt}(b'_{st}, b'_{lt})b'_{lt}$ as well.

Under these bond prices, it is optimal to set $\boldsymbol{B}(\Omega) = (B_{st}(\Omega), B_{lt}(\Omega)) = 0$. By assumption the borrower is never a net saver. Hence, the borrower either chooses $\boldsymbol{B}(\Omega) = 0$ or defaults today. In the former case $D(y_H, \boldsymbol{B}(\Omega)) = D(y_L, \boldsymbol{B}(\Omega)) = 0$, while in the latter case $D(\Omega) = 1$. If $y = y_L$, the proof is done. Otherwise, one can repeat the same argument backward as $D(y_H, b_{st}, b_{lt}) = 1$ implies $D(y_L, b_{st}, b_{lt}) = 1$.

– Part III

Consider two cases. First, assume that $V^P_{NB}(y_H, b_{st}, b_{lt}) - V^D(y_H) < V^P_{NB}(y_L, b_{st}, b_{lt}) - V^D(y_L)$. Given this, there is no buyback in the transition from y_H to y_L. Instead buybacks arise in the transition from y_L to y_H when $V^P_{NB}(y_L, b_{st}, b_{lt}) - V^D(y_L)$ is small enough. I therefore need to rule out buybacks after a sufficiently long series of y_L. Assume by contradiction that this is the case. In the period with y_L right before the buyback, $\mathbb{E}[V(\Omega')] = \mathbb{E}[V^D(y')]$. This is because a buyback arise if either y_L or y_H realizes next period. Given this, the borrower chooses the largest level of debt compatible with $\chi < \infty$ next period. Denote such level of debt by $\underline{b}_{lt} < 0$. This means that borrower never actually crosses $V^D(y_L)$. A buyback is therefore suboptimal. If the borrower does not buyback tomorrow, it can issue today $b'_{lt} < \underline{b}_{lt}$. The borrower is strictly better off since it consumes strictly more today and $\mathbb{E}[V(\Omega')] \geq \mathbb{E}[V^D(y')]$ anyway. There is therefore no buyback in y_L.

Second, assume that $V^P_{NB}(y_H, b_{st}, b_{lt}) - V^D(y_H) \geq V^P_{NB}(y_L, b_{st}, b_{lt}) - V^D(y_L)$. In this case, a buyback is optimal in y_H only if it is optimal in y_L. I use here a modified version of Chatterjee and Eyigungor (2012, Proposition 2). Fix $\chi < \infty$ and denote the consumption under no buyback in y_H by c^{NB} and the underlying bond policy vector by $\boldsymbol{B}^{NB} = (b'_{st}, b'_{lt})$. Denote the value of the outstanding bond portfolio next period by $v^{NB}_B = b'_{st} + b'_{lt}(1 + \delta q_{lt}(b''_{st}, b''_{lt}))$. Consider any buyback resulting to $v^B_B \geq v^{NB}_B$ and denote the corresponding bond policy $\boldsymbol{B}^B$ and consumption c^B in y_H. Notice that $v^B_B \geq v^{NB}_B$ is a weaker requirement than $\boldsymbol{B}^B \geq \boldsymbol{B}^{NB}$. If a buyback is not optimal in y_H, then

$$u(c^{NB}) + \beta\mathbb{E}[V(y', \boldsymbol{B}^{NB})] \geq u(c^B) + \beta\mathbb{E}[V(y', \boldsymbol{B}^B)]. \tag{A.4}$$

Since $\mathbb{E}[V(y', \boldsymbol{B}^{NB})] \leq \mathbb{E}[V(y', \boldsymbol{B}^B))]$ from Proposition A.1, it holds that $c^{NB} \geq c^B$.[16] Define $\Delta = c^{NB} - c^B \geq 0$ and denote the consumption under no buyback in y_L borrowing $\boldsymbol{B}^{NB}$ by $\tilde{c}^{NB}$. It holds that $\tilde{c}^{NB} < c^{NB}$ given that $y_L < y_H$. By strict concavity of $u(\cdot)$, one has that

$$u(\tilde{c}^{NB}) - u(\tilde{c}^{NB} - \Delta) > u(c^{NB}) - u(c^{NB} - \Delta),$$

where $\tilde{c}^{NB} - \Delta = \tilde{c}^B$ and $c^{NB} - \Delta = c^B$. Then (A.4) implies

$$u(\tilde{c}^{NB}) + \beta\mathbb{E}[V(y', \boldsymbol{B}^{NB})] > u(\tilde{c}^B) + \beta\mathbb{E}[V(y', \boldsymbol{B}^B)].$$

Since $\boldsymbol{B}^B$ corresponds to any buyback such that $v^B_B \geq v^{NB}_B$, a buyback cannot be optimal in y_L when it is not optimal in y_H. However, $V^P_{NB}(y_H, b_{st}, b_{lt}) - V^D(y_H) \geq V^P_{NB}(y_L, b_{st}, b_{lt}) - V^D(y_L)$ implies the reverse. Hence, there is no buyback in y_L.

Notice that fixing $\chi < \infty$ is without loss of generality. In this proof, I fix χ and adapt $\boldsymbol{B}^B$ but I could do the opposite. Moreover, a borrower would never conduct a buyback resulting to $v^B_B < v^{NB}_B$. In this case it is preferable to enter only the primary market and avoid the payment of χ.

□

[16] It directly follows from equation (A.2) in Proposition A.1.

A.4 Proof of Lemma 1

Lemma 1 (Permanent autarky)**.** *Permanent autarky is an equilibrium.*

Proof. Suppose that the lenders believe that $D(\Omega) = 1$ for all Ω. Under permanent autarky, consumption is simply y. Following Proposition 3 Part II, under $\mathbb{E}[D(\Omega')] = 1$ for all (b'_{st}, b'_{lt}), it is optimal to set $\boldsymbol{B}(\Omega) = (B_{st}(\Omega), B_{lt}(\Omega)) = 0$. By assumption the borrower is never a net saver. This implies that $c(\Omega) = y + b_{st} + b_{lt}(1 + \delta q_{lt}(\boldsymbol{B}(\Omega))) < y$ and

$$u(c(\Omega)) + \sum_{t=1}^{\infty} \beta^t \pi(y_t) u(y_t) < u(y) + \sum_{t=1}^{\infty} \beta^t \pi(y_t) u(y_t) = V^D(y),$$

when $b_{st} + b_{lt}(1 + \delta q_{lt}(\boldsymbol{B}(\Omega))) < 0$. It is therefore optimal for the borrower to default confirming the lenders' beliefs. □

A.5 Proof of Proposition 4

Proposition 4 (Constrained Efficient Allocation)**.**

I. (Efficiency). *$V^l(y, x)$ is strictly decreasing, while $V^b(y, x)$ is strictly increasing in $x \in \tilde{X} \equiv [x_D(y_L), \overline{x}]$ for all $y \in Y$ and $x_D(y_H) > x_D(y_L)$.*

II. (Risk-Sharing). *$c(y_L, x) < c(y_H, x)$ and $x'(y_L, x) < x'(y_H, x)$ for $x < x_D(y_H)$ and $c(y_L, x) = c(y_H, x)$ and $x'(y_L, x) = x'(y_H, x)$ otherwise. Also, $c(y_L, x_D(y_L)) = y_L$ and $c(y_H, x_D(y_H)) < y_H$.*

III. (Liabilities). *$V^l(y_L, x) < V^l(y_H, x)$ for all $x \in \tilde{X}$.*

Proof. I prove each part of the proposition one by one:

– Part I

The law of motion of the relative Pareto weight is given by $x'(y) = (1 + \nu(y))\eta x$, while the first-order condition on consumption reads $u_c(c(y)) = \frac{1}{(1+\nu(y))x} = \frac{\eta}{x'(y)}$.

Consider the interval $[x_D(y_L), \overline{x}]$. From the law of motion of the relative Pareto weight, x' is strictly increasing in x. From the first-order conditions on consumption, c is strictly increasing in x. Hence, so does the value of the borrower. In opposition, with a greater c (or equivalently a greater x), the instantaneous payoff of the lenders, $y - c$, decreases. That is the lenders' value is strictly decreasing in x.

Moreover, as $V^D(y_H) > V^D(y_L)$ and the value of the borrower is strictly increasing in x, it must be that $x_D(y_H) > x_D(y_L)$.

– Part II

Observe that, given the first-order condition, $c(y_L, x) \leq c(y_H, x)$ only when $\nu(y_L) \leq \nu(y_H)$. Assume by contradiction that $\nu(y_L) > \nu(y_H)$. This implies that $c(y_L, x) > c(y_H, x)$ and $x'(y_L, x) > x'(y_H, x)$. Given this, from Part I, $V^b(y', x'(y_L, x)) > V^b(y', x'(y_H, x))$. As a result.

$$u(c(y_L, x)) + \beta \mathbb{E} V^b(y', x'(y_L, x)) > u(c(y_H, x)) + \beta \mathbb{E} V^b(y', x'(y_H, x)).$$

Moreover as $\nu(y_L) > \nu(y_H) \geq 0$

$$u(c(y_H, x)) + \beta \mathbb{E} V^b(y', x'(y_H, x)) \geq V^D(y_H),$$
$$u(c(y_L, x)) + \beta \mathbb{E} V^b(y', x'(y_L, x)) = V^D(y_L).$$

This implies that $V^D(y_L) > V^D(y_H)$, a contradiction. Hence, $\nu(y_L) \leq \nu(y_H)$ which gives $c(y_L, x) \leq c(y_H, x)$ and $x(y_L, x) \leq x(y_H, x)$ as desired.

Furthermore, by definition, when $x \geq x_D(y_H)$, then $\nu(y) = 0$ for all y implying that $c(y_L, x) = c(y_H, x)$ and $x(y_L, x) = x(y_H, x)$. Otherwise, $c(y_L, x) < c(y_H, x)$ and $x(y_L, x) < x(y_H, x)$.

Finally, observe that if $x = x_D(y_L)$, then $V^b(y_L, x_D(y_L)) = V^D(y_L)$ given the definition of $x_D(y_L)$ and $V^b(y_H, x_D(y_L)) = V^D(y_H)$ given that $x_D(y_L) < x_D(y_H)$. As a result, $\mathbb{E}[V^b(y', x_D(y_L))] = \mathbb{E}[V^D(y')]$ implying that $c(y_L, x_D(y_L)) = y_L$. In addition, if $x = x_D(y_H)$, then $V^b(y_H, x_D(y_H)) = V^D(y_H)$ given the definition of $x_D(y_H)$ and $V^b(y_L, x_D(y_H)) > V^D(y_L)$ since $x_D(y_H) > x_D(y_L)$. As a result, $\mathbb{E}[V^b(y', x')] > \mathbb{E}[V^D(y')]$ implying that $c(y_H, x_D(y_H)) < y_H$.

– Part III

This proof is a modified version of Thomas and Worrall (1990, Lemma 4). The value of liabilities in the optimal contract is given by

$$V^l(y, x) \equiv y - c(y, x) + \frac{1}{1+r} \mathbb{E} V^l(y', x'(y, x)).$$

Assume by contradiction that for a given x it holds that $V^l(y_H, x) \leq V^l(y_L, x)$. Consider the pooling allocation in which $u(\ddot{c}(y_H, x)) = u(\ddot{c}(y_L, x)) = u(c(y_H, x))$ and $\ddot{V}^b(y_H, x) = \ddot{V}^b(y_L, x) = V^b(y_H, x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to $\ddot{V}^l(y_H, x) > \ddot{V}^l(y_L, x)$ which is a direct contradiction. Hence, $V^l(y_H, x) > V^l(y_L, x)$.

□

A.6 Proof of Proposition 5

Proposition 5 (Existence and Uniqueness). *Under Assumption 1, given initial conditions* (y_0, x_0)*, there exists a unique constrained efficient allocation.*

Proof. Existence and uniqueness follow from Theorem 3 in Marcet and Marimon (2019). The two authors make the following assumptions: A1 a well defined Markov chain process for y, A2 continuity in $\{c\}$ and measurability in y, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lenders and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given my environment. Since feasible c is bounded, payoffs functions are bounded as well. This combined with the fact that the outside options are also bounded ensure that A4 is met. Whether A3 is satisfied depends on the initial condition (y_0, x_0). Assumption 1 ensures feasibility and that the strict interiority condition is satisfied.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding to the original contract problem (12). To obtain the recursive formulation of the contract, I have normalized the co-state variable. I relied on the the homogeneity of degree one in (μ_b, μ_l) to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if I define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}_+^2\}$ and the set of feasible consumption by $A = \{c \in \mathbb{R}_+\}$, the correspondence $SP : A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Marcet and Marimon (2019) additionally show that the the saddle point functional equation (14) is a contraction mapping. Thus, given the strict concavity assumptions of $u(\cdot)$, the allocation is unique. □

A.7 Proof of Proposition 6

Proposition 6 (Inverse Euler Equation). *The inverse Euler equation is given by*

$$\mathbb{E}\left[\frac{1}{u_c(c(y'))(1+\nu(y'))}\right] = \eta \frac{1}{u_c(c(y))},$$

Proof. The law of motion of the relative Pareto weight is given by $x'(y) = (1 + \nu(y))\eta x$ and the level of consumption by $u_c(c(y)) = \frac{1}{x(1+\nu(y))}$. Isolating x leads to

$$x = \frac{1}{u_c(c(y))(1+\nu(y))}. \tag{A.5}$$

Plugging this back into the law of motion gives $x'(y) = (1+\nu(y))\eta\frac{1}{u_c(c(y))(1+\nu(y))}$. Replacing $x'(y)$ by with the forward equivalent of (A.5) gives $\frac{1}{u_c(c(y'))(1+\nu(y'))} = \eta\frac{1}{u_c(c(y))}$. Taking expectations on both sides gives the inverse Euler equation. □

A.8 Proof of Proposition 7

Proposition 7 (Ergodic Set)**.** *The ergodic set of relative Pareto weights* $[x^{lb}, x^{ub}] \subset \tilde{X}$ *is such that* $x'(y_H, x^{ub}) = x^{ub}$*,* $x'(y_L, x^{lb}) = x^{lb}$ *with* $x^{lb} = x_D(y_L) < x^{ub} < x_D(y_H)$*.*

Proof. The law of motion of the relative Pareto weight $x'(y) = (1+\nu(y))\eta x$ is dictated by the relative impatience, η, and the binding participation constraint, ν. Given that $\eta < 1$, the relative Pareto weight increases only if $\nu(y) > 0$ is sufficiently large to overcome impatience. By definition, when $x \geq x_D(y)$, $\nu(y,x) = 0$ meaning that impatience eventually dominates the limited commitment. Hence, impatience prevents the contract to reach $x_D(y_H)$ as $\nu(y_H, x_D(y_H)) = 0$. This implies that $x_D(y_H) > x^{ub}$. Moreover, $x'(y_L, x) < x'(y_H, x)$ when $x < x_D(y_H)$ implying that $x^{ub} > x^{lb}$. Finally, as $x'(y)$ is bounded below by $x_D(y_L)$, $x^{lb} = x_D(y_L)$. Said differently, $x^{lb} < x_D(y_L)$ would violate (11).

To show the existence of a unique ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as any point in the interval $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \rightarrow \mathbb{R}$ as

$$Q(x, G) = \sum_{y'} \pi(y')\mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of y_L, the relative Pareto weight transit to x^{lb} through impatience. Hence for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(y_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M = 1$ y_H, the relative Pareto weight transit to x^{ub} through the binding participation constraint meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}]) = \pi(y_H) > 0$. Setting $\epsilon = \min\{\pi(y_L)^M, \pi(y_H)\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. □

A.9 Proof of Lemma 2

Lemma 2 (Long-Term Bond Price)**.** *Under* (16)*, the long-term bond price is the unique fixed point of* $\bar{q}_{lt}$*, is decreasing and*

$$\frac{1+\delta\chi}{1+r-\delta} > \bar{q}_{lt}(x'(y_L,x)) \geq \bar{q}_{lt}(x'(y_H,x)) \geq \frac{1}{1+r-\delta},$$

with strict inequality if $\bar{b}_{lt}(x'(y_L,x)) < 0$.

Proof. Recall that the long-term bond price is given by

$$\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}\mathbb{E}\left[(1-\bar{D}(y',x'))\left\{1+\bar{M}(y',x')\delta\chi+\delta\bar{q}_{lt}(y',x')\right\}\right], & \text{if } b_{lt}(x) < 0 \\ \frac{1}{1+r}\mathbb{E}\left[1+\delta\bar{q}_{lt}(y',x')\right], & \text{else} \end{cases}$$

I consider that $\bar{D}(y',x') = 0$ for all (y',x') and $\bar{M}(y',x') = 1$ if $y' = y_H$ as well as $x' < x^{ub}$ and $\bar{M}(y',x') = 0$ otherwise. From Proposition 7, y_H and $x \leq x^{ub}$ arises with strictly positive probability for any (y,x),

$$\frac{1+\chi}{1+r-\delta} > \bar{q}_{lt}(y,x) \geq \frac{1}{1+r-\delta}.$$

I implicitly assume that the buyback is feasible when it arises. Define $\mathcal{Q}$ as the space of bounded functions $\bar{q}_{lt} : [\underline{x},\bar{x}] \to [0, \frac{1+\delta\chi}{1+r-\delta}]$ and $\mathbb{T} : \mathcal{Q} \to \mathcal{Q}$ as

$$\mathbb{T}\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}(\pi(y_L)[1+\delta\bar{q}_{lt}(x'(y_L,x))] + \pi(y_H)[1+\delta\chi+\delta\bar{q}_{lt}(x'(y_H,x))]) & \text{if } x < x^{ub} \\ \frac{1}{1+r}\sum_{y'}\pi(y')[1+\delta\bar{q}_{lt}(x'(y',x))] & \text{else} \end{cases}$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$. Moreover, a closer inspection $\bar{q}_{lt}$ indicates that it is decreasing. This implies that $\bar{q}_{lt}(x'(y_H,x)) \leq \bar{q}_{lt}(x'(y_L,x))$ as $x'(y_H,x) > x'(y_L,x)$ for all x in the above specified domain. The inequality is strict whenever $b_{lt}(x'(y_L,x)) < 0$ given the specification of the buyback. Assume by contradiction that there exists a x such that $\bar{q}_{lt}(x'(y_H,x)) = \bar{q}_{lt}(x'(y_L,x))$ and $b_{lt}(x'(y_L,x)) < 0$. This requires that there exists a subset of $[x^{lb},x^{ub}]$ where $\bar{q}_{lt}$ stays constant. The contradiction is immediate as $x'(y_H,x) = x_D(y_H)$, whereas $x'(y_L,x) < x_D(y_H)$ for any $x \in [x^{lb},x^{ub}]$. □

B Alternatives to Buybacks

In this section, I provide alternatives to buybacks: excusable defaults, variable-coupon bonds and variable-maturity bonds.

First, Grossman and Van Huyck (1988) develop the concept of excusable defaults. The idea is that defaults which are on the path of play agreed by all market participants are not punished. In other words, the debt contract specifies *ex ante* the circumstances in which the borrower is allowed to repudiate its debt without suffering from markets exclusion. Given this, if defaults were excusable, then the borrower's binding participation constraint – i.e. $x = x^{lb}$ – could be interpreted as a default. The issue is that the borrower might be willing to repudiate debt more often than what the debt contract specifies. To deal with this, one can use endogenous borrowing limits as in the main analysis. Nevertheless, the concept of excusable defaults has little empirical relevance. The closest policy that has been implemented to this date is a sovereign debt standstill analyzed by Hatchondo et al. (2020) with the only difference that there is no arrears accumulation in excusable defaults.[17] In addition, Mateos-Planas et al. (2023) show that if the borrower were to choose the conditions for excusable defaults, such events would be extremely rare if not inexistent.

Second, the long-term debt can have a variable coupon as in Faraglia et al. (2019) and Aguiar et al. (2021). Particularly, assume that the coupon payment is a choice variable, say $\kappa \in [0, 1]$, for the borrower. Obviously, the variability of the coupon is a covenant in the debt contract. In other words, changes in the coupon are agreed by the contracting parties *ex ante* and do not pertain to a contract renegotiation – e.g. an outright default in case of a reduced coupon payment. With such debt contract, it is possible to implement the constrained efficient allocation in two ways: the borrower sets a standard coupon payment $\tilde{\kappa}$ and either increases it to $\bar{\kappa} > \tilde{\kappa}$ when $y = y_H$ and $x < x^{ub}$ or decreases it to $\underline{\kappa} < \tilde{\kappa}$ when $y = y_L$ and $x = x^{lb}$. In the former case, the borrower is not willing to pay a larger coupon payment and the lenders ought to have enough market power to enforce this payment. In opposition, in the case of reduced coupon payment, the borrower might be tempted to reduce the coupon payment more frequently than the Planner would. Thus, the lenders would also need to supervise the coupon policy.

Lastly, bonds can have variable maturities. That is, the maturity of outstanding short-term (long-term) debt can be lengthened (shortened). Similar to variable-coupon bonds, this is a feature which should be explicitly mentioned in the debt contract. To implement the constrained efficient allocation, the borrower ought to either lengthen the maturity of short-term debt when $y = y_H$ and $x < x^{ub}$ or shorten the maturity of long-term debt when

[17] See also the recent proposal on contingent convertible bonds by Hatchondo et al. (2022).

$y = y_L$ and $x = x^{lb}$. Implicitly, by shortening the maturity, the borrower pays less coupons than it initially promised. In other words, the claim of legacy creditors is reduced. The opposite happens in the case of maturity lengthening. Thus, similar to variable-coupon bonds, maturity lengthening would need to be enforced, while maturity shortening should be closely supervised to avoid lowering legacy creditors' claim too frequently.

C Data

Table C.1 specifies the source of the data used in the analysis. For GDP data, I rely on OECD Quarterly National Accounts. I detrend the logarithm of the GDP series using the HP filter with a smoothing parameter of 1600. Spreads data come from the Global Financial Database.

Table C.1: Data Sources and Definitions

Series	Sources	Unit
Output	OECD Quarterly National Accounts[a]	volumes estimates
Global bonds	Refinitiv Eikon Datastream[b]	current USD
Global bonds	Brazilian National Treasury[c]	current USD
Buybacks	Brazilian National Treasury[d]	current USD
EMBI+	Global Financial Database[e]	basis point

[a] Real GDP, real GDP YoY growth. Subject: B1_GE. Measure: VPVOBARSA, GYSA.
[b,c] USD-denominated bonds issued by the government of Brazil. See Table C.2.
[d] Buyback by month and by bond in financial and face value. See Monthly Debt Report.
[e] Government bond spread. Series code: EMBPBRAD.

Regarding the buybacks there are three data sources. First, I use the Monthly Debt Report published by the Brazilian National Treasury to retrieve the amount bought back for each bond. Reports of buyback are bimonthly and specify the face and the financial value of each bond repurchased.[18] Note that three reports are missing: January-February 2009, September-October 2013 and January-February 2015. In addition, the 2006 report is annual with only the total financial value across the different bonds available. When missing, I estimate the financial value using the average yield to maturity for each bond. Note that I do not account for bonds denominated in foreign currency other than USD.[19]

Second, the Brazilian National Treasury publishes the list of all foreign denominated bonds issued by the Brazilian government since 1995. The list contains the coupon rate, the

[18] Starting 2018, buybacks reports are monthly.

[19] There were some bonds in EUR and JPY involved in the buyback program. However, they correspond to a negligible amount compared to the ones denominated in USD.

issuance yield, the issuance and maturity date, the ISIN code, the number of re-openings and the volumes in USD.

Table C.2: Bonds

Bond	ISIN Code	Issuance	Maturity	Re-Opening	Coupon	Total Issuance	Total Buyback
BR01	US105756AD24	1996	2001	0	8.9	0.8	0.0
BR04	XS0049988636	1999	2005	2	11.6	3.0	0.0
BR05	US105756AS92	2001	2005	0	9.6	1.0	0.0
BR06	US105756AQ37	2001	2006	0	10.3	1.5	0.0
BR07	US105756AM23	2000	2007	2	11.3	1.5	0.5
BR07B	US105756AW05	2003	2007	0	10.0	1.0	0.4
BR08	US105756AG54	1998	2008	0	9.4	1.3	0.3
BR08B	US105756AU49	2002	2008	0	11.5	1.3	0.6
BR09	US105756AJ93	1999	2009	0	14.5	2.0	0.9
BR09F	US105756BC32	2004	2009	0	-	0.8	0.3
BR10	US105756AV22	2002	2010	0	12.0	1.0	0.5
BR10N	US105756BA75	2003	2010	0	9.3	1.5	0.6
BR11	US105756AY60	2003	2012	3	10.0	1.3	0.7
BR12	US105756AT75	2002	2012	0	11.0	1.3	0.5
BR13	US105756AX87	2003	2013	0	10.3	1.3	0.6
BR14	US105756BD15	2004	2014	2	10.5	1.3	0.8
BR15	US105756BG46	2005	2015	3	7.9	2.1	1.0
BR16	US105756BJ84	2005	2016	0	12.5	1.5	0.0
BR17	US105756BM14	2006	2017	3	6.0	2.5	0.5
BR18	US105756BH29	2005	2018	0	8.0	4.5	2.9
BR19	US105756BE97	2004	2019	2	8.9	1.5	0.9
BR19N	US105756BQ28	2009	2019	3	5.9	2.3	0.1
BR20	US105756AK66	2000	2020	0	12.8	1.0	0.5
BR21	US105756BS83	2010	2021	4	4.9	3.0	0.3
BR22	US105756BL31	2006	2022	3	12.5	1.4	0.0
BR23	US105756BU30	2012	2023	2	2.6	2.2	0.0
BR24	US105756AR10	2001	2024	0	8.9	2.2	0.9
BR24-BRL	US105756BT66	2012	2024	0	8.5	1.7	0.0
BR24B	US105756AZ36	2003	2024	0	8.9	0.8	0.7
BR25	US105756BF62	2005	2025	2	8.8	2.2	1.3
BR25	US105756CD06	2020	2025	2	2.9	1.8	0.0
BR25	US105756BV13	2013	2025	2	4.3	4.3	0.0
BR26	US105756BX78	2016	2026	2	6.0	2.5	0.3
BR27	US105756AE07	1997	2027	2	10.1	3.5	1.4
BR28	US105756BZ27	2017	2028	0	4.6	3.0	0.0
BR28-BRL	US105756BN96	2007	2028	5	10.3	2.5	0.0
BR29	US105756CA66	2019	2029	2	4.5	2.0	0.0
BR30	US105756AL40	2000	2030	2	12.3	1.6	0.7
BR30-1a	US105756CC23	2020	2030	2	3.9	3.5	0.0
BR30-2a	US105756AL40	2000	2030	2	12.3	1.6	0.0
BR31-1a	US105756CG37	2023	2031	0	6.3	2.0	0.0
BR31-2a	US105756CE88	2021	2031	0	3.8	1.5	0.0
BR32	US105756CK49	2024	2054	0	6.1	2.0	0.0
BR33	US105756CF53	2023	2033	0	6.0	2.2	0.0
BR34	US105756BB58	2004	2034	4	8.3	2.7	0.9
BR34A	US105756CH10	2024	2034	0	6.1	2.2	0.0
BR37	US105756BK57	2006	2037	5	7.1	3.0	0.4
BR40	US105756AP53	2000	2040	0	11.0	5.2	3.8
BR41	US105756BR01	2009	2041	3	5.6	2.9	0.1
BR45	US105756BW95	2014	2045	0	5.0	3.6	0.0
BR47	US105756BY51	2016	2047	2	5.6	3.0	0.0
BR50	US105756CB40	2019	2050	3	4.8	4.0	0.0
BR54	US105756CJ75	2024	2054	0	7.1	2.2	0.0

Note: The table depicts all the USD-denominated bonds issued by Brazil. Total issuance and total buyback are denominated in USD billion. For the bond name $-1a$ and $-2a$ indicate the bond tranche, $-BRL$ indicates that either the principal or the coupon payment is in BRL. The other letters were attributed by the Brazilian National Treasury.

Third, I use Refinitiv Eikon Datastream to find all the bonds published by the Brazilian National Treasury. This enables me to obtain the complete history of bond prices and the yields to maturity as well as more information on the bond structure. Given this and the second dataset, I can re-construct the cashflow stream of each bond. Table C.2 indicates all the USD-denominated bonds issued by the Brazilian government. Recall that I compute the buyback premium only for bonds which where issued prior to 2006.