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On a lender of last resort with a central bank and a stability Fund ☆

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ABSTRACT

We explore the complementarity between a central bank and a financial stability Fund in stabilizing sovereign debt markets. The central bank pursuing its mandate can intervene with public sector purchasing programs, buying sovereign debt in the secondary market, provided that the debt is safe. The sovereign sells its debt to private lenders, through market auctions. Furthermore, it has access to a long-term state-contingent contract with a Fund: a country-specific debt-and-insurance contract that accounts for no-default and no-over-lending constraints. The Fund needs to guarantee gross-financial-needs and no-over-lending. We show that these constraints endogenously determine the 'optimal debt maturity' structure that minimizes the Required Fund Capacity (RFC) to make all sovereign debt safe. However, the Fund may have limited absorption capacity and fall short of its RFC. The central bank may be able to cover the difference, in which case there is perfect complementarity and the joint institutions act as an effective 'lender of last resort'. We calibrate our model to the Italian economy and find that with a Fund contract its 'optimal debt maturity' is 2.9 years with an RFC of 90% of GDP, which is above what the European Stability Mechanism (ESM) could reasonably absorb, but may be feasible with an ECB *Transmission Protection Instrument* (TPI) intervention. In contrast, the average maturity of Italian sovereign debt has been circa 6.2 years, with a needed absorption capacity of around 105% of GDP, which may call for a maturity restructuring to ease the activation of TPI.

1. Introduction

Regulatory institutions tend to be successful when they address specific market failures in a way that does not stifle the markets and discourage innovation but rather makes them more robust. The Fed itself was founded in response to the panic of 1907 when the

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need for a lender of last resort for solvent but illiquid institutions and responsive monetary policy became an obvious compelling public policy issue. Thomas Cooley, 2009.[1]

In his statement in the U.S. House of Representatives, Thomas Cooley provided a role and a definition of what a *lender of last resort* (LOLR) should be and should avoid. It should address the specific market failure of having *solvent but illiquid institutions*, which immediately raises a non-trivial issue: the LOLR should be able to discriminate between solvency and illiquidity crises and support those, and only those, institutions that are solvent. It should avoid interfering with the market – as its name, *lender of 'last' resort* indicates,– and should also avoid supporting institutions that do not contribute to the growth of the economy, or distort it with moral hazard problems. Cooley's definition goes back to the "Lombard Street" ideas of Bagehot (1873) and forward to the application of the mechanism design approach to the design of *lender of last resort* institutions.[2] This paper belongs to this move forward. In the context of an economic union, the main institutions that may need a LOLR are the sovereign states, with a key market failure being to have sustainable sovereign debts which cannot be rolled over when *belief-driven* Self-Fulfilling Crises (SFCs) emerge.

We build on Ábrahám et al. (2022) and Liu et al. (2022), who show how a well-designed Financial Stability Fund can effectively stabilize sovereign debt, provided it can offer long-term state-contingent contracts which account for non-default and a no-over-lending constraints.[3] While in the former the Fund absorbs all the debt, with an exclusivity contract, in the latter the Fund contract complements the funding of private lenders. In particular, in any period and state, the Fund first takes the lead in announcing what should be the maximum credit line for the next period, consistent with an 'optimal' government policy (consumption, employment, debt or savings and insurance), as well as its corresponding policy. If the announcement is credible, the amount of debt the Fund needs to absorb can be relatively small and debt is safe, becoming an important supply of safe assets. However, both papers assume – as in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), et al. – that sovereign countries can commit to next-period payments, which helps the credibility of the announcement and the deterrence of *belief-driven* SFCs (Ayres et al., 2018). They also assume that the Fund can always absorb all the newly issued debt if needed and that the maturity structure of the debt is given.[4] We dispense with these assumptions by modeling an economy where: first, borrowing countries cannot commit to their payments, as in Cole and Kehoe (2000) and Aguiar et al. (2020). Indeed, it is standard practice by Treasuries to hold an auction for a given amount of debt without guaranteeing that it will succeed with the final price of a safe debt. Second, the Fund may not have the capacity to absorb all the debt foreseen in its contracts. Third, the borrower decides the maturity of the debt, accounting for the Fund design.

A first novel result is that the Fund averts both fundamental *and* belief-driven debt crises. In the latter, the Fund stands ready to provide the necessary resources in case of failed private debt auction, similar to Roch and Uhlig (2018). In the former, the Fund debt-and-insurance contract is contingent on exogenous shocks, as well as on the endogenous non-default constraint. Without default, there is no debt-dilution; as, for example, in Aguiar and Amador (2020). As a result, sovereign countries always remain in the 'safe zone'. Furthermore, the Fund also accounts for a no-over-lending constraint. This constraint, in a union, prevents undesired permanent transfers (debt mutualization) and, with private lenders, internalizes an externality that they do not account for: excessive lending. The absence of both default and over-lending allows a constrained-efficient allocation of consumption and labor but requires a Fund's *minimum intervention* in terms of debt absorption. A main implication of this first result is that, in the decentralized formulation of the economy with the Fund, the results of Liu et al. (2022) on the existence and uniqueness of a *Recursive Competitive Equilibrium*, as well as the Second and First Welfare Theorems, concerning the constrained-efficient allocation, extend to our economies.

A second result is that in assessing the *absorption capacity* of the Fund, the maturity of the debt matters. In particular, if at the steady-state, self-fulfilling equilibria need to be prevented, then the Fund must, on the one hand, be able to cover the necessary Gross Financial Needs (GFN) of the consolidated public sector when new debt is issued – a requirement which is decreasing with maturity; on the other hand, if the no-over-lending constraint binds in equilibrium, then the Fund's *minimum intervention* needs to absorb part of the sovereign debt – a requirement which is increasing in maturity. Therefore, 'there is an optimal maturity structure' that minimizes the *absorption capacity* that is needed for any given Fund contract. We call the corresponding minimal absorption the *Required Fund Capacity* (RFC). To the best of our knowledge, this is the first determination of the 'optimal maturity structure' as a result of an institutional design.

1 Statement of Thomas F. Cooley, Professor of Economics, Stern School of Business New York University, for the Hearing on The Role of the Federal Reserve before The House Financial Services Committee, Subcommittee on Domestic Monetary Policy U.S. House of Representatives, July 9, 2009.

2 Cooley was very aware that these requirements – what 'should be and should avoid'– were very demanding, and better be implemented by a proper institution, with independence being a necessary but not sufficient condition. For example, at the same time that he praised the original Fed of 1907, he was critical of how it was pursuing its role as LOLR one hundred years later: "It's kind of shocking, when we think about it, that we provide so much insurance to financial institutions without charging any insurance premium. The fact that the Federal Reserve is the LOLR to financial institutions is part of the bedrock of central banking since the days of Walter Bagehot. But that's changed. The idea that you would lend on good collateral at a penalty rate of interest has long since gone by the board. It's now the *lender of first resort* on possibly dodgy collateral and motivated entirely by keeping the liquidity of the system." Interview with Steve Forbes, Chairman and Editor-in-Chief of Forbes Media. May 27, 2009. See also Acharya et al. (2011) and Acharya et al. (2009).

3 Ábrahám et al. (2022) also account for moral hazard problems, which can also be incorporated in our framework.

4 Furthermore, Liu et al. (2022) show that if the no-over-lending constraint is binding in the steady-state, the optimal maturity is one period.

It should be noted that the RFC is needed to make the Fund's policy credible. In this case, private lenders with rational expectations have those anchored and debt is safe. In other words, the RFC is needed as a *prudential institutional design*, even though the GFN do not need to be absorbed by the Fund in equilibrium. Furthermore, the RFC is also a *robust institutional design* since we also allow for private lenders to disbelieve the Fund announcement, in which case the minimal absorption may be needed to prevent SFCs.

Thirdly, if the Fund does not have enough *absorption capacity* to be able to take all the new sustainable debt of a sovereign along the development of the contract, the Central Bank (CB) might step in 'to purchase whatever else is needed'.[5] In this case, the CB policy is effective and there are neither SFCs – that is, there is no failed debt auction – nor over-lending. There is a *perfect complementarity between the Fund and the CB* if together they have the necessary *absorption capacity*: the Fund guarantees that, with the backup of the CB, debt is sustainable and the auction is successful, which allows the CB to activate its debt purchasing programme without taking risks, making the Fund policy credible.

We calibrate our economy to the Italian economy since Italy is a large country in the euro area, and has been the only 'stressed country' during the euro crisis which has not asked for – and, therefore, not received – any official lending support, e.g. from the European Stability Mechanism (ESM) or the International Monetary Fund (IMF). Furthermore, we can take, as a reference, two related calibrations of the Italian economy: Bocola and Dovis (2019) and Liu et al. (2022). For a given Italian maturity structure, in a Fund contract, belief-driven sudden stops of private lending can arise in steady state and the no-over-lending constraint can bind. Then, we obtain that for Italy the average 'optimal maturity' is 2.9 years, while it has been 6.2 years on average in the data. This means that while the RFC is roughly 90% of GDP, the needed *absorption capacity* given the average maturity of Italian debt is 105%. In other words, even if Italy were to adopt the 'optimal maturity' the RFC may be too large for the ESM but may be feasible if, in coordination, the European Central Bank (ECB) would activate its Transmission Protection Instrument (TPI) or Outright Monetary Transaction (OMT) programmes. The calibration also allows us to estimate the welfare gains of having a LOLR in this context. The consumption-equivalent welfare gains at the steady-state are significant but lower than what would be if there were not a risk of SFCs, even if its likelihood is – close to the estimates of Bocola and Dovis (2019) – very small. These, and related quantitative results, form our fourth contribution.

The paper is organized as follows. Section 2 briefly discusses the related literature. Section 3 describes the environment of the economies under study, which can either be without the Fund, Section 4, or with the Fund, Section 5 and, for the latter, Section 6 shows that there is an optimal maturity structure. Then, Section 7 introduces the Central Bank and discusses its intervention which can result in a *perfect complementarity* with the Fund. Section 8 presents the quantitative analysis and Section 9 concludes.

2. Literature review

This paper contributes to different strands of the literature: sovereign debt and default, optimal maturity structure of sovereign debt, dynamic contract design, and official lending and LOLR.

Broadly speaking, the quantitative sovereign default literature features two types of models. On the one hand, building on the seminal work of Eaton and Gersovitz (1981), the workhorse models of Aguiar and Gopinath (2006) and Arellano (2008) consider sovereign defaults driven by changes in fundamentals.[6] The ability to commit to intra-period issuance means that the government can choose along the price schedule offered by the market, ruling out the possibility of multiplicity à la Calvo (1988).[7] On the other hand, Cole and Kehoe (2000) and Aguiar et al. (2020) introduce the possibility of rollover-crises and consider sovereign defaults driven not only by changes in fundamentals but also by changes in beliefs.[8] As shown by Ayres et al. (2018), a key difference between those two types of models is the timing of actions. Lenders' beliefs matter if the sovereign's decision to default occurs after the auction takes place, giving rise to the possibility of failed auctions. We contribute to this strand of the literature by characterizing the role of a stability Fund which acts as a LOLR in the presence of both fundamental and belief-driven debt crises. Our work, therefore, relates to Roch and Uhlig (2018) who develop a bailout agency with minimal intervention. The main difference is that our Fund is constrained efficient and prevents not only belief but also fundamental-driven debt crises. We then apply our analysis to the case of Italy similar to Bocola and Dovis (2019). Different from them, our focus is to characterize the RFC and on the counterfactual welfare gains for Italy if, together with the ECB absorption capacity, there had been a LOLR. Furthermore, since with the LOLR, there is no default, there is no *debt dilution* either, and therefore the economy is also immune to the SFCs characterized by Aguiar and Amador (2020). That is the LOLR keeps the economy in the safe zone, always. The need for fiscal capacity for a government to act as LOLR is also present in Bocola and Lorenzoni (2020).

The maturity structure of sovereign debt has been studied in different settings. A first strand of literature addresses the maturity choice absent incentive to default. Angeletos (2002) and Buera and Nicolini (2004) determine the optimal maturity structure under commitment and incomplete markets. The maturity structure determines how the available assets span

5 Following the principle that a CB cannot make risky asset purchases, such an intervention requires that sovereign market turbulence – disruptive for monetary policy – is not a reflection of the issued debt not being sustainable. This is an explicit requirement in the new Transmission Protection Instrument (TPI) of the European Central Bank (ECB). See Section 7 for a more detailed discussion.

6 See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

7 See Lorenzoni and Werning (2019) for a discussion and application of this type of multiplicity.

8 See also Conesa and Kehoe (2017) and Corsetti and Maeng (2021).

shocks; hence, it could be chosen to provide insurance to the fiscal authority.[9] This motive is absent in our case as the sovereign can enter insurance contracts directly with the Fund. Lucas and Stokey (1983) study optimal fiscal policy with complete markets, but lack of commitment. The government has an incentive to manipulate the risk-free real interest rate, by changing taxes which affects the lenders' marginal utility, something ruled out by our small open economy framework with risk-neutral lenders. Their main result is that the maturity of debt should be flat, resembling the issuance of consols. Extending the analysis to the case of incomplete markets, Debortoli et al. (2017) show there is a trade-off between the insurance motive and the incentive to manipulate the interest rate, which goes in favor of having a nearly flat maturity structure.

Accounting for default risk, Kiiashko (2022) shows that the optimal maturity is decaying in the environment of Lucas and Stokey (1983).[10] Besides this, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that long-term debt provides an incentive to dilute existing creditors in the framework of Eaton and Gersovitz (1981). Arellano and Ramanarayanan (2012) and Niepelt (2014) study the optimal maturity choice in the same environment and find it optimal to shorten maturities as default risk increases. This is because there is a trade-off between the desire to hedge fiscal risk by extending maturities and the need to prevent debt dilution incentives. Hatchondo et al. (2016) measure the effect of debt dilution by calibrating their model to Spain, and study debt covenants that can mitigate these effects. We contribute to this strand of literature by unveiling a new trade-off. On the one hand, similar to Cole and Kehoe (2000), longer maturities limit the occurrence of SFCs shrinking the crisis zone. On the other hand, longer maturities hinder the intervention of official LOLRs, because of the required debt absorption when there is a risk of over-lending. We show that the optimal maturity which minimizes the size of the LOLR is the one balancing those two forces. This provides a new criterion to determine the maturity of sovereign debt in the presence of SFCs and official lenders. As we said, to the best of our knowledge, this is the first paper that studies a maturity structure determined by an institutional constraint.

Our study is also related to the literature on optimal contracts with limited enforcement constraints such as Kehoe and Levine (2001), Kocherlakota (1996) and, in particular, Kehoe and Perri (2002) and Restrepo-Echavarria (2019) who already applied the Lagrangian-recursive approach developed by Marcet and Marimon (2019). Unlike Aguiar et al. (2019) and Aguiar and Amador (2020), our Planner's problem accounts for all creditors in both the objective function and the constraint set and integrates two-sided limited enforcement constraints. Our decentralization relies on the approach of Alvarez and Jermann (2000), while our focus is close to Thomas and Worrall (1994) who already studied international lending contracts, with one-sided limited commitment. Finally, as we already said, our work more closely contributes to the recent literature on the design of an optimal stability Fund by Ábrahám et al. (2022) and Liu et al. (2022), but in contrast with them we also incorporate the model of Cole and Kehoe (2000) in our benchmark economy to explicitly account for SFCs. Unlike Roch and Uhlig (2018), we show that the intervention of the Fund completely eliminates both fundamental and belief-driven debt crises, provided that the financial capacity of the Fund is large enough. In that logic, we consider the intervention of a Central Bank (CB) to complement the Fund's intervention. Finally, unlike, the contemporaneous work of Dovis and Kirpalani (2023), who also study a Financial Stability Fund as a LOLR in economies with defaultable sovereign debt, our Fund is essential not only to prevent defaults but also to implement the unique constrained-efficient allocation which the private economy cannot attain otherwise. Furthermore, also in contrast with them, our Fund never incurs undesired expected losses and our Recursive Constrained Equilibria are not only *ex-ante* constrained-efficient but also *ex-post*.

Building a financial stability Fund able to stabilize sovereign debt and preventing the occurrence of debt crises addresses the large literature regarding the IMF and other international institutions lending practices.[11] Our framework goes beyond the simple catalytic role of official assistance. The Fund's contract preserves the incentive compatibility requirements for a LOLR outlined by Bagehot (1873) by removing the trade-off between moral hazard risks and crisis prevention evaluated in Gete and Melkadze (2020).

The LOLR function of the combined Fund-CB closely relates to the debate on the need to develop the Fiscal Union within the European Economic and Monetary Union (EMU) and the interplay between fiscal and monetary policies.[12] Indeed, as shown in Bianchi and Mondragon (2022), in a monetary union the inability to use monetary policy for macroeconomic stabilization amplifies the severity of the recession in case of a roll-over crisis making lenders more prone to run. Different from the recent literature that focus on the trade-off between price stability and fiscal stabilization,[13] we explore the complementarity between a CB and a Fund, when the Fund has limited resources. In fact, the inability of the Fund and of the CB to be backed by a single Treasury which has access domestic resources – a key difference with the Federal Reserve system – raises the same issues outlined by Cecchetti (2014) for an international LOLR. These considerations motivate the analysis of the impact of constraints on the capacity of the Fund and of the CB to fully act as LOLR. As noted by

[9] Whether or not this is feasible in practice remains unclear. Buera and Nicolini (2004) found the position required to hedge cyclical shifts in the slope of the yield curve to be implausibly large.

[10] See also Dovis (2019) and Wicht (2023) for the optimal maturity under default risk.

[11] See for instance Corsetti et al. (2006), Morris and Shin (2006) and Rochet and Vives (2010).

[12] See Marimon and Cooley (2018) and Marimon and Wicht (2021) for a discussion on how our Fund proposal relates to this literature and it can be implemented within EMU.

[13] See for example Corsetti and Dedola (2016) on the role of monetary policy to prevent SFCs or Bianchi et al. (2022) in the context of the fiscal theory of the price level.

Whelan (2022), the capacity of the Eurosystem to hold sovereign bonds might be subject to hard limits on the basis of legal considerations and constraints.

3. Environment

In this section, we present the general environment of the model. We first start with the preference and the technology before defining the market structure and the timing of actions.

3.1. Preference and technology

The environment is as in Liu et al. (2022). We assume an infinite-horizon small open economy with a single homogenous consumption good in discrete time. There is a sovereign acting as a representative agent and taking decisions on behalf of the small open economy, a Fund acting as an official lender that can offer long-term contracts providing credit and insurance, and a continuum of private lenders. The Fund and the private lenders have access to international financial markets – with a risk-free rate of r – and are risk-neutral. Regarding the sovereign, preferences over consumption and leisure are represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, where $\beta \in (0,1)$ is the discount factor, n_t is labor, $1-n_t$ leisure and c_t consumption at time t. The sovereign is relatively impatient such that $\beta < 1/(1+r)$. We adopt a specific form of utility function to obtain a (stochastic) balanced growth path and to simplify the expression of utility in terms of detrended consumption: $U(c,n) = u(c) + h(1-n) = \log(c) + \xi \frac{(1-n)^{1-\zeta}}{1-\zeta}$. The sovereign has access to a labor technology $y = \theta f(n)$ subject to decreasing returns to scale, where $f'(n) > 0$, $f''(n) < 0$. Moreover, θ represents a shock to productivity growth. The law of motion of the shock is given by $\theta_t = \gamma_t \theta_{t-1}$, where γ_t represents the growth rate at time t. We denote the history of θ up to time t by θ^t. The exact form of the growth shocks is detailed in Section 8.

3.2. Debt and sustainability

The sovereign country has access to a long-term state-contingent contract with the Fund – a credit-insurance line that we specify below – and debt contracts with a continuum of competitive private lenders. Private lenders' contracts are a continuum of simple long-term debt contracts, which we assume to have a common maturity and coupon (δ, κ); i.e. given a private lending portfolio of value $b_{l,t}$, $(1-\delta)b_{l,t}$ matures in period t while $\delta\kappa b_{l,t}$ is the coupon payment private lenders must receive from the country for the non-maturing debt.[14]

In contrast, the design of the Fund contract is based on a risk assessment of the country which, as it is common practice in *debt sustainability analysis* (DSA), also accounts for the effect of the same Fund contract in enhancing the sustainability of the country's sovereign debt. The Fund contract is a state-contingent asset, $a_{l,t}$, with the aforementioned maturity and coupon. It can be decomposed into a debt, $\bar{a}_{l,t}$, and insurance components, $\hat{a}_{l,t}(\theta^t)$; that is, $a_{l,t}(\theta^t) = \bar{a}_{l,t}(\theta^{t-1}) + \hat{a}_{l,t}(\theta^t)$.

As in Liu et al. (2022), we assume that the private lenders cannot credibly provide insurance due to *strategic complementarity* in their actions. More precisely, if a private lender believes that future private lenders are unwilling to provide insurance, it will be itself unwilling to provide insurance as the sovereign will eventually default. As a result, the Fund is essential as the private economy cannot reach the constrained efficient allocation without it.

The private lenders' and Fund's contracts establish that at time t and state-history θ^t the country must transfer $\tau_f(\theta^t)$ for its state-contingent liabilities with the Fund and $\tau_p(\theta^t)$ for its non-contingent debt liabilities with the private lenders. We denote $\tau(\theta^t) \equiv \tau_f(\theta^t) + \tau_p(\theta^t)$ as the total transfer the country pays. That is, given a consumption and employment plan $\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}$, in period-state (t, θ^t) feasibility implies that

$$\tau(\theta^t) = \theta_t f(n(\theta^t)) - c(\theta^t); \tag{1}$$

that is, $\tau(\theta^t)$ is the *primary surplus* in state-history θ^t. The expected present value of future transfers discounted with the risk-free rate r should cover the outstanding amount of debt and debt-insurance liabilities.

$$\mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_p(\theta^j) \geq b_{l,t}, \quad \text{and} \quad \mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_f(\theta^j) \geq a_{l,t}, \tag{2}$$

where $b_{l,t}$ and $a_{l,t}$ represent the value of the debt and debt-insurance *lent* at (t, θ^t) by the private lenders and the Fund, respectively. The sum of the two is denoted by $\omega_{l,t} = b_{l,t} + a_{l,t}$, and we will also denote by $\bar{\omega}_{l,t} = b_{l,t} + \bar{a}_{l,t}$, when we only account for the *debt* liabilities.

The sovereign can default on its liabilities. We assume that the sovereign's liabilities with the Fund – i.e. $-a_l$ – have no seniority with respect to the sovereign debt in the hands of other agents.[15] Under default, the sovereign suffers from an

[14] We denote by b_l bond held by the private lenders, while we denote by $-b$ bond issued by the sovereign. By market clearing, $b = -b_l$. That is, $b > 0$ denotes an asset and $b < 0$ denotes a debt from the point of view of the sovereign. We adopt the same notation for all other securities.

[15] Liu et al. (2022) show that with a well-designed Fund, whether it has seniority or not is irrelevant from the perspective of the sovereign and private lenders, although with seniority the Fund will need to have a larger *absorption capacity* than without seniority; therefore, seniority is not an advantage for the Fund, as it is usually considered.

output penalty $\theta^D \leq \theta$ and loses access to both the private bond market and the Fund. Later, it can reintegrate the private bond market with some probability, λ, but cannot obtain the assistance of the Fund anymore.

3.3. Timing and beliefs

We assume that there is a shock $\rho \in \{0, 1\}$ that coordinates private creditor beliefs over defaults when there is uncertainty regarding those happening. The process $\{\rho_t\}$ is i.i.d. and independent of the process $\{\gamma_t\}$. In other words, while θ is a fundamental shock, ρ is a non-fundamental one (usually labelled a sunspot). The variable ρ signals whether the private lenders, if they coordinate, roll over the non-maturing debt in states where these decisions are uncertain. Particularly, if the success of the auction depends on the lenders' coordination on $\rho = 0$: if $\rho = 0$, the roll-over takes place (i.e. $b_{l,t+1} \geq \delta b_{l,t}$), whereas if $\rho = 1$, the roll-over does not take place (i.e. $b_{l,t+1} \leq \delta b_{l,t}$). We denote by ϱ the probability that $\rho = 1$.

The timing of actions is the following. At the beginning of a given period, the fundamental shock θ realizes and the Fund announces what it considers to be the appropriate lending policy to the sovereign borrower $\{\omega_{l,t+1}(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$. Knowing the new realization of θ the sovereign may default. If not, decides its prospective borrowing $b_{l,t+1}$ and if $b_{l,t+1} - \delta b_{l,t} > 0$ sets an auction. Afterwards, the non-fundamental shock ρ realizes, the auction takes place and the sovereign decides to default if the auction fails. Finally, if there has not been default, the Fund and the sovereign implement their part of the Fund contract $(\bar{a}_{l,t}, \{\hat{a}_{l,t+1}(\theta^{t+1})\}_{\theta^{t+1}|\theta^t})$. After this contractual phase, or default, production and consumption take place.

Note that with this timing, the Fund is a proper LOLR as it can provide resources to the sovereign if the private bond auction turns out to be unsuccessful; in fact, the Fund contract prevents default and, therefore, its contract is implemented.

4. The economy without the Fund

In this section, we define the economy without the Fund following Cole and Kehoe (2000). We first expose the sovereign's problem before defining the crisis zone in which SFCs occur. This economy will later be the basis of our calibration for Italy.

4.1. The sovereign's problem

The state space in the economy without the Fund is (s, b) where $s \equiv (\theta, \rho)$, although θ is observed before ρ. The sovereign's overall beginning of the period value is given by

$$
\begin{aligned}
&V(\theta, b) = \varrho V((\theta, 1), b) + (1-\varrho) V((\theta, 0), b), \text{ with} \\
&V(s, b) = \max_{b'} \left\{ V^P(s, b, b'), V^D(s) \right\}, \qquad (3)
\end{aligned}
$$

where V^P and V^D correspond to the value of repayment and default, respectively. Under repayment, the value of the sovereign for a given (s, b, b') reads

$$
\begin{aligned}
V^P(s, b, b') = \max_{c,n} \; & U(c, n) + \beta \mathbb{E}\left[V(s', b') \Big| s \right] \\
\text{s.t.} \quad & c + q_p(s, b, b')(b' - \delta b) \leq \theta f(n) + (1 - \delta + \delta\kappa) b.
\end{aligned}
$$

Under default, the sovereign is excluded from the financial markets for some time and receives an output penalty. Given that production and consumption take place after default, or the debt-insurance contracts are set, when there is default – either fundamental or not – the sovereign defaults in all its liabilities and, therefore, the value under default is independent of ρ and for a given s reads

$$
V^D(s) = V^D(\theta) = \max_n \; U(\theta^D f(n), n) + \beta \mathbb{E}\left[(1-\lambda) V^D(s') + \lambda V(s', 0) \Big| s \right],
$$

where $\theta^D \leq \theta$ entails the output penalty and λ is the probability of re-accessing the market. Note that, similar to Eaton and Gersovitz (1981), the value of default only depends on θ and not on (ρ, b, b'). In particular, note that with $\varrho \in (0, 1)$, $V(\theta, b) = V^D(\theta)$ if, and only if, $V((\theta, 1), b) = V((\theta, 0), b) = V^D(\theta)$. The autarky value V^D will later represent the sovereign's outside option in the Fund.

From (3), we can specify two sovereign policies. First, we can define $b' = B(s, b)$ as the optimal private bond policy. Second, we can define the default policy $d(s, b, b')$ which takes value one if default is optimal (i.e. $V^P(s, b, b') < V^D(s)$) and zero otherwise. Moreover, given $B(s, b)$, we can set $D(s, b) \equiv d(s, b, B(s, b))$.

4.2. Self-fulfilling debt crises

Given that private lenders are risk neutral and break even in expectation, the price of one unit of private bond is given by

$$q_p(s,b,b') = \frac{1-d(s,b,b')}{1+r}\left[1-\delta+\delta\kappa+\delta\mathbb{E}\left[(1-d(s',b',b''))q_p(s',b',b'')\middle|s\right]\right]. \tag{4}$$

The fact that $d(s,b,b')$ appears in the price equation is the main difference between Eaton and Gersovitz (1981) and Cole and Kehoe (2000). In the former, private lenders are repaid before the new debt is issued. That is upon repayment, $d(s,b,b') = 0$ for all (s,b,b'), although $\mathbb{E}d(s',b',b'')$ may not be zero. In contrast, in Cole and Kehoe (2000), the decision to repay comes after the issuance of new debt meaning that $d(s,b,b')$ is not necessarily zero for all (s,b,b').

This gives rise to multiplicity in equilibria. To better understand this, first define the fundamental price at which debt would be traded if $d(s,b,b') = 0$,

$$q_p^f(s,b,b') = \frac{1}{1+r}\left[1-\delta+\delta\kappa+\delta\mathbb{E}\left[(1-d(s',b',b''))q_p(s',b',b'')\middle|s\right]\right]. \tag{5}$$

This corresponds to the price in Eaton and Gersovitz (1981). Given this, we can then separate the state space (s,b) in three zones:

1. The safe zone:
   In this region, the sovereign repays (i.e. $D(s,b) = 0$). This is understood by private lenders (with rational expectations) and, therefore, the realization of ρ is irrelevant ('private lenders do not bother to look for sunspots'). The private bonds are priced according to (5).
2. The default zone:
   In this region, the sovereign defaults (i.e. $D(s,b) = 1$) and, as in the safe zone, the realization of ρ is irrelevant.
3. The crisis zone:
   In this region there is uncertainty about whether the sovereign debt auction will succeed. If the private lenders expect it will not (i.e. $\rho = 1$) then they abstain, the auction fails, and the sovereign defaults (i.e. $D(s,b) = 1$), while if the private lenders expect the auction will succeed (i.e. $\rho = 0$) they actively participate and the sovereign repays (i.e. $D(s,b) = 0$). In particular, define

   $$\mathcal{C}(\varrho) \equiv \left\{s,b \mid \text{ default is optimal if } q(s,b,b') = 0, \text{ but } q(s,b,b') > 0 \text{ is possible}\right\},$$

   where ϱ is the probability that $\rho = 1$. In $\mathcal{C}(\varrho)$, $V^P(s,b,b') \geq V^D(s) \geq V^P(s,b,\delta b)$, i.e. if $q(s,b,b') = 0$. Thus, in this region, the private bond price is given by

   $$q_p(s,b,b') = \begin{cases} 0 & \text{if } \rho = 1 \text{ and } b' \leq \delta b \\ q_p^f(s,b,b') & \text{else} \end{cases}$$

Note that, in fact, the characterization of the three zones only depends on (θ, b) and ϱ, although the latter is a fixed parameter in our simulations. In particular, in the default zone the sovereign may as well decide to default at the beginning of the period, while *belief-driven* default can only happen after $\rho = 1$ has been revealed. As we will show in the next section, the Fund's intervention enables the sovereign to always stay in the safe zone.

5. The economy with the Fund

In this section, we derive the economy with the Fund. We first present the Fund contract before decentralizing it and characterizing its intervention.

5.1. The Fund contract

The Fund contract is the outcome of a Planner's problem with two agents — the sovereign and the Fund itself — taking into account the participation of a continuum of private lenders in absorbing credit needs. This defines an allocation, of consumption and employment, which the Fund takes as the benchmark policy the sovereign will follow, and the corresponding transfers of the sovereign to the lenders. Conditional on the debt absorbed by the private lenders, the part that the Fund contract absorbs is determined. As we will see below, the latter has an insurance and potentially a debt component.

Formally, we say that $\{c(s^t), n(s^t)\}_{t=0}^{\infty}$ is a solution to the Fund's contacting problem in *sequential form*, given $b_{l,0}$, if there exist sequences of transfers $\{\tau_p(s^t), \tau_f'(s^{t+1})\}_{t=0}^{\infty}$, with associate $\{b_{l,t}\}_{t=0}^{\infty}$ satisfying (2), such that:

$$\max_{\{c(s^t),n(s^t)\}_{t=0}^{\infty}} \mathbb{E}\left[\mu_{b,0}\sum_{t=0}^{\infty}\beta^t U(c(s^t),n(s^t)) + \mu_{l,0}\sum_{t=0}^{\infty}\left(\tfrac{1}{1+r}\right)^t \tau(s^t)\middle|s_0\right] \tag{6}$$

$$\text{s.t. } \mathbb{E}\left[\sum_{j=t}^{\infty}\beta^{j-t}U(c(s^j),n(s^j))\middle|s^t\right] \geq V^D(s^t), \tag{7}$$

$$\mathbb{E}\left[\sum_{j=t}^{\infty}\left(\tfrac{1}{1+r}\right)^{j-t}\tau(s^j)\middle|s^t\right] \geq \theta_{t-1}Z + b_{l,t} \tag{8}$$

$$\tau(s^t) \equiv \tau_f(s^t) + \tau_p(s^t) = \theta(s^t) f(n(s^t)) - c(s^t), \tag{9}$$
for all $(t, s^t), t \geq 0$, with $\mu_{b,0}$, $\mu_{l,0}$ given.

The sovereign consumes $c(s^t)$ and provides labor $n(s^t)$. The initial Pareto weights of the sovereign and the lenders – respectively $\mu_{b,0}$ and $\mu_{l,0}$ – are given by the initial break-even condition for the Fund

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \tau(s^t) \middle| s_0\right] = \theta_{-1} Z - b_0.$$

Equations (7) and (8) represent the *limited enforcement constraints* of the borrower and the Fund, respectively. The borrower's outside option is to default and is given by $V^D(s^t)$ as defined in Section 4. The underlying assumption is that if the sovereign defaults from the Fund, it also defaults on its private debt liabilities and then is never allowed to return to the Fund in the future.

The limited enforcement constraint of the Fund depends on $Z \leq 0$ and b_l. The former variable indicates the level of redistribution of the Fund. To prevent that the Fund provides permanent transfers to a sovereign – e.g. to prevent debt mutualization – we will assume that $Z = 0$, i.e. that in no state the Fund contract has expected losses.[16] Similarly, b_l indicates the level of outstanding private debt the sovereign needs to repay. In that logic, larger b_l tightens the constraint. As shown by Liu et al. (2022), the constraint (8) implies that in states where the sovereign's future surpluses might not cover an additional amount of debt both lenders provide less resource to avoid losses that would go beyond the contract's terms. Thus, the limited enforcement constraint of the Fund internalizes a pecuniary externality that competitive private lenders usually do not: the fact that marginal lending can be excessive. As a result, we interpret (8) as a no-over-lending constraint.

As already noted by Liu et al. (2022), with $\tau(s^t) \equiv \tau_f(s^t) + \tau_p(s^t)$ in (9), the Fund contract accounts for both the private lenders and the Fund. In other words, it takes into consideration the sovereign's entire debt position. While the Fund *directly* specifies $\tau_f(s^t)$ taking as given $\tau_p(s^t)$, effectively the contract is taking into account the total surplus $\tau_f(s^t) + \tau_p(s^t)$ when evaluating the limited enforcement constraints, since only in this way it is capable of consistently stabilizing the sovereign's entire debt position. An equivalent interpretation is that the Fund stands ready to absorb the debt position of the sovereign in the form of private bonds, when necessary. In such circumstances, there is complete credit (risk) transfer from the private bond investors to the Fund, up to certain limits implied by the participation constraints both from the Fund and the sovereign. We come back to this later.

Using the approach of Marcet and Marimon (2019), we can express (6) in recursive form. Appendix A presents all the details of such exposition. The key element in achieving a recursive formulation is to keep track of an additional state variable: the relative Pareto weight $x = \frac{\mu_b}{\mu_l}$, where μ_b and μ_l are the accumulated values of the Lagrange multipliers attached to the *forward-looking* constraints (7) and (8), respectively. The Fund contract problem (6) has a solution as long as, for any state s, there are rents to share, this means that at most one of these two constraints can bind. The relative Pareto weight is a sufficient statistic for the contract history as it evolves according to the binding constraints (7) and (8). Particularly, it increases when the sovereign's constraint binds and decreases when the Fund's constraint binds. Larger x means that the contract gives more weight to the sovereign relative to the lenders. This translates into larger consumption and larger continuation value of the sovereign. The opposite happens when x decreases.

5.2. The decentralized Fund contract

Following the work of Alvarez and Jermann (2000) and Krueger et al. (2008), we can decentralize the Fund contract to obtain the asset position and the price related to the Fund's transfer. As before, the financial market is composed of private lenders and the Fund.

The sovereign has two funding opportunities: long-term defaultable bonds, b', on the private bond market at a unit price of $q_p(s, \omega, \bar{\omega}')$ and $|\Theta|$ state-contingent securities with the Fund $a'(\theta')$ at a unit price of $q_f(\theta', \omega'(\theta')|s, \omega)$.[17] Moreover, the portfolio $a'(\theta')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q_f(s, \omega, \bar{\omega}') \equiv \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)$, and an insurance portfolio of $|\Theta|$ Arrow securities $\hat{a}'(\theta')$. Thus we have that $a'(\theta') = \bar{a}' + \hat{a}(\theta')$ with $\bar{a}' = \frac{\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega) a'(\theta')}{q_f(s, \omega, \bar{\omega}')}$ and $\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)\hat{a}'(\theta') = 0$ which represents the market clearing condition of Arrow securities.

At the start of a period, the sovereign holds $a = \bar{a} + \hat{a}$ in the Fund and b in the private bond market which together sum to an entire position – including insurance and debt – of $\omega = a + b$ and a total *debt* position of $\bar{\omega} = \bar{a} + b$. Given this, the transfer to the Fund and the private lenders are respectively:

[16] Alternatively, we could have had $Z(s^t) \leq 0$ to allow for within the period solidarity in some crisis states, resulting in permanent transfers (e.g. the outbreak of a pandemic). To simplify the analysis we postulate that at no state should be expected losses; however, to properly define the conditional expectation when the right hand side of (8) is predetermined by s_{t-1}, we need to keep track of θ_{t-1} since $\theta_t = \gamma_t \theta_{t-1}$, in our growth formulation where γ_t is the shock at the beginning of period t.

[17] Notice that $a'(\theta')$ is not contingent on ρ.

$$\tau_f(s) = \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)(a'(\theta') - \delta a(\theta)) - (1 - \delta + \delta\kappa)a(\theta), \text{ and}$$
$$\tau_p(s) = q_p(s, \omega, \bar{\omega}')(b' - \delta b) - (1 - \delta + \delta\kappa)b.$$

Under the above market structure, the sovereign's problem reads

$$W^b(s, a, b) = \max_{\{c,n,b',\{a'(\theta',b')\}_{\theta'\in\Theta}\}} U(c, n) + \beta\mathbb{E}\left[W^b(s', a'(\theta', b'), b')\middle|s\right] \tag{10}$$
$$\text{s.t. } c + \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)(a'(\theta', b') - \delta a) + q_p(s, \omega, \bar{\omega}')(b' - \delta b)$$
$$\leq \theta f(n) + (1 - \delta + \delta\kappa)(a + b)$$
$$\omega'(\theta') = a'(\theta', b') + b' \geq \mathcal{A}_b(\theta'). \tag{11}$$

Equation (11) is the equivalent of the participation constraint (7) whose purpose is to prevent the occurrence of defaults. The endogenous borrowing limit $\mathcal{A}_b(\theta')$ is therefore such that

$$W^b(s', \ddot{a}'(\theta', \ddot{b}'), \ddot{b}') = V^D(s') \text{ for all } \ddot{a}'(\theta', \ddot{b}') + \ddot{b}' = \mathcal{A}_b(\theta').$$

Private lenders are competitive and risk-neutral financial intermediaries. Their maximization problem is given by

$$W^p(s, a_l, \bar{a}_p, b_l) = \max_{\{c_p, b_l', \bar{a}_p'\}} c_p + \frac{1}{1+r}\mathbb{E}\left[W^p(s', a_l', \bar{a}_p', b_l')\middle|s\right] \tag{12}$$
$$\text{s.t. } c_p + q_p(s, \omega, \bar{\omega}')(b_l' - \delta b_l) + q_f(s, \bar{a}_p, \bar{a}_p')(\bar{a}_p'(\theta') - \delta\bar{a}_p) \leq (1 - \delta + \delta\kappa)(b_p + \bar{a}_p).$$

The private lenders also have access to the bonds issued by the Fund. This enables the bond price in the Fund and in the private bond market to coincide through arbitrage. Besides this, the trade of private bonds satisfies the following transversality condition

$$\lim_{n\to\infty} \mathbb{E}\left\{\left[\prod_{j=0}^n Q_p\left(s^{t+j}, \bar{\omega}\left(s^{t+j}\right), \bar{\omega}\left(s^{t+j+1}\right)\right)\right]b_l\left(s^{t+j+1}\right)\middle|s^t\right\} = 0, \quad \text{with} \tag{13}$$
$$Q_p\left(s^{t+j}, \bar{\omega}\left(s^{t+j}\right), \bar{\omega}\left(s^{t+j+1}\right)\right) = \frac{q_p\left(s^{t+j}, \bar{\omega}\left(s^{t+j}\right), \bar{\omega}\left(s^{t+j+1}\right)\right)}{1 - \delta + \delta\kappa + \delta q_p\left(s^{t+j+1}, \bar{\omega}\left(s^{t+j+1}\right), \bar{\omega}\left(s^{t+j+2}\right)\right)}. \tag{14}$$

Note that we will consider equilibria where, without loss of generality, $\bar{a}_p = 0$, therefore we simplify notation by eliminating $\bar{a}_p$ if not necessary. Notably, we write $W^p(\theta, a_l, 0, b_l) \equiv W^p(\theta, a_l, b_l)$.

An important object which emanates from (12) is the private lending policy, $b_l' = B_l(s, a_l, b_l)$ which is taken as given by the Fund whose maximization problem is given by

$$W^f(\iota, a_l, b_l) = \max_{\{c_f, \{a_l'(\theta', b_l')\}_{\theta'\in\Theta}\}} c_f + \tfrac{1}{1+r}\mathbb{E}\left[W^f(\iota', a_l'(\theta', b_l'), b_l')\middle|\iota\right] \tag{15}$$
$$\text{s.t. } c_f + \textstyle\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)(a_l'(\theta', b_l') - \delta a_l) \leq (1 - \delta + \delta\kappa)a_l,$$
$$\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' \geq \mathcal{A}_f(\theta', b_l'), \tag{16}$$
$$\text{with } b_l' = B_l(s, a_l, b_l) \text{ given,}$$

where $\iota \equiv \{\theta^-, \gamma, \rho\}$. Note that in (16), $\omega_l'(\theta')$ and $a_l'(\theta', b_l')$ are simultaneously determined for a given b_l'. That is, the Fund, as security trader, choosing $a_l'(\theta', b_l')$ determines $\omega_l'(\theta')$ by (16); alternatively, the Fund could have chosen $\omega_l'(\theta')$ and use (16) to determine $a_l'(\theta', b_l')$. The variable $\mathcal{A}_f(\theta', b_l)$ represents an endogenous limit defined as

$$W^f(\iota', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z. \tag{17}$$

Adding equations (17) to the value of the lender (12) and applying the transversality condition (13), we obtain

$$W^f(\iota', \mathcal{A}_f(\theta', b_l') - b_l', b_l') + W^p(s', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z + b_l'.$$

This gives the decentralized counterpart of the lenders' participation constraint in (8),

$$W^l(\iota', a_l'(\theta', b_l'), b_l') \equiv W^f(\iota', a_l'(\theta', b_l'), b_l') + W^p(s', a_l'(\theta', b_l'), b_l') \geq \theta Z + b_l'. \tag{18}$$

As we consider $\bar{a}_p = 0$, the market clearing condition in the Fund is given by $a(\theta, b) + a_l(\theta, b) = 0$ for all (s, b). In addition, the initial asset holdings of the sovereign in the Fund, $a(\theta_0, b_0) = -a_l(\theta_0, b_0) = 0$, are given. We can now define a *Recursive Competitive Equilibrium* (RCE) in this environment as follows.

Definition 1 *(Recursive Competitive Equilibrium (RCE)).* Given value functions for the outside value options of the sovereign, $V^D(s)$, and of the lenders, $\theta^- Z + b_l$, a *Recursive Competitive Equilibrium (RCE)* consists of: prices $q_f(\theta', \omega'(\theta')|s, \omega)$ and $q_p(s, \omega, \bar{\omega}')$; value functions $W^b(s, a, b)$, $W^f(\iota, a_l, b_l)$, and $W^p(s, a_l, b_l)$; endogenous limits, $\mathcal{A}_b(\theta')$ and $\mathcal{A}_f(\theta', b_l')$, and policy functions $c(s, a, b)$, $c_f(\iota, a_l, b_l)$, $c_p(s, a_l, b_l)$, $n(s, a, b)$, $a'(s', b') = A(\theta', s, a, b, b')$, $a_l'(\iota', b_l') = A_l(\iota', \iota, a_l, b_l, b_l')$, $b' = B(s, a, b)$ and $b_l' = B_l(s, a_l, b_l)$, which are solutions to the problems of the sovereign, private lenders and the Fund, and all markets clear. In particular, the announcement $\omega_l'(\theta')$ is equal to its equilibrium value; i.e. $\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' = -\omega'(\theta')$.

Using the fact that the borrowing constraints of the borrower and the lenders do not bind at the same time, the price is determined by the agent whose constraint is not binding (Krueger et al., 2008). It then follows that

$$q_f(\theta', \omega'(\theta')|s, \omega) = \tfrac{\pi(\theta'|\theta)}{1+r}\Big[(1-\delta+\delta\kappa) + \delta \textstyle\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|s', \omega')\Big] \max\Big\{\tfrac{u_c(c')}{u_c(c)}\eta, 1\Big\}. \tag{19}$$

Given the above price schedule, the intertemporal discount factor is defined by

$$Q_f\big(\theta', \omega'(\theta')|s, \omega\big) \equiv \frac{q_f\big(\theta', \omega'(\theta')|s, \omega\big)}{1-\delta+\delta\kappa+\delta\sum_{\theta''|\theta'} q_f\big(\theta'', \omega''(\theta'')|s', \omega'\big)}.$$

The implicit interest rate in the Fund is then defined by $r_f(s, \omega, \bar{\omega}') \equiv \frac{1}{Q_f(s,\omega,\bar{\omega}')} - 1$ with $Q_f(s, \omega, \bar{\omega}') \equiv \sum_{\theta'|\theta} Q_f\big(\theta', \omega'(\theta')| s, \omega\big)$.

Provided that the private lenders have access to the Fund's securities, no arbitrage is possible between the Fund and the private bond market for the sovereign. Hence, the bond prices in the Fund and the private bond market are alike unless private lenders coordinate on $\rho = 1$.

Furthermore, from (19), observe that when (18) does not bind, $r_f(s, \omega, \bar{\omega}') = r$ meaning that securities are traded at the risk-free rate. Conversely, when (18) binds in a specific state s', $r_f(s, \omega, \bar{\omega}') < r$ meaning that securities are traded below the risk-free rate. In other words, a *negative spread* appears. The next subsection explains why this is the case.

The *negative spread* generates a wedge between the lenders' discount factor and the pricing kernel (19). That is why the valuation equation (2) holds with inequality. In Particular, this means that the Fund can still provide insurance (i.e. $\hat{a}_l(\theta) < 0$) while (18) being binding.

5.3. The Fund's intervention

We first derive the Fund's dynamic relative to the private bond market, before characterizing the Fund's intervention in terms of default insurance.

There are two types of sudden stops in this model: fundamental and belief-driven. The former is a consequence of the Fund's intervention, while the latter owes to the coordination of the private lenders' beliefs as exposed in Section 4. In the absence of such sudden stops, the division of $\bar{\omega}_l'$ between b_l' and $\bar{a}_l'$ is indeterminate as $q_f(s, \omega, \bar{\omega}') = q_p(s, \omega, \bar{\omega}')$ through arbitrage. It is then without loss of generality that we can set $\bar{a}_l' = 0$. That is the Fund solely provides Arrow securities. In contrast, in the presence of sudden stops, the Fund needs to absorb some level of debt, i.e. $\bar{a}_l' > 0$.

A fundamental-driven sudden stop arises when lending becomes excessive. That is when (18) binds, meaning that if there is further lending the sovereign's future surpluses will not cover an additional amount of debt. From the previous subsection, we know that in this situation, $r_f(s, \omega, \bar{\omega}') < r$. This negative spread is a signal sent by the Fund to the market participants that further lending will become excessive. Given that $\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|s, \omega)\hat{a}_l'(\theta') = 0$, the negative spread restricts the Fund's lending to avoid losses that would go beyond the contract's terms. In addition, it causes private lenders to stop lending to the sovereign as the rate of return settles below r.[18] As a result, a binding constraint (18) not only restricts the Fund's lending to the sovereign, it also sustains a no-trade equilibrium in the private bond market. Importantly, this sudden stop is driven by the fact that lending becomes excessive and is independent of the sunspot realization.

The Fund's intervention during a fundamental-driven sudden stop must be able to account for the sovereign's liabilities with the Fund and backup those with the private lenders. Note that, from (2), when (18) binds, we have $\theta^- Z + b_l' \geq \omega_l'(\theta')$ with $b_l' \leq \delta b_l$ due to the lending sudden stop. Given that $\omega_l'(\theta') = \bar{\omega}_l' + \hat{a}_l'(\theta')$, the maximal amount of debt the Fund may have to absorb is $\theta^- Z - \min\big\{\hat{a}_l'(\theta') : (18) \text{ binds } \wedge\ \pi(\theta'|\theta) > 0\big\} + \delta b_l$. That is, with respect to the Fund, the solidarity component $Z \leq 0$ and the complement to the maximal amount of insurance the sovereign may receive with positive probability. Regarding private lenders, the Fund has to guarantee a maximal absorption of δb_l. In other words, the Fund must stand ready to guarantee just enough lending for the sovereign to honor its long-term liabilities. This is because the private lending sudden stop endangers the ability of the sovereign to maintain the value of its long-term debt, either directly — under the counterfactual interpretation that each period the sovereign buys and sells the long-term debt — since it may not be able to borrow from the private lenders to cover it; or, indirectly since private lenders may want to sell their

[18] The private lenders are willing to *borrow* from the Fund in terms of a portfolio of securities which constitutes risk-free asset a_p, and investing the funds to earn a risk-free rate r. Nevertheless, the binding constraint of the Fund also prevents such trading activities.

holdings of over-priced, low-return, long-term debt in exchange for safe assets. The Fund's guarantee can therefore amount up to δb_l.

A belief-driven sudden stop arises when the economy is in the crisis zone and $\rho = 1$. There, private lenders believe that the sovereign will be unable to repay and only offer $b_l' \leq \delta b_l$. Given this, the sovereign's participation constraint (11) might not hold which implies that there is a risk of default. To avoid this, the Fund has to substitute private lending. In other words, it needs to provide the resource the private lenders would have offered if $\rho = 0$ in addition to its own supply of insurance and credit. Thus, given a policy announcement ω_l, the Fund's intervention is a commitment to absorb $\bar{a}_l' \geq \bar{\omega}_l - \delta b_l$ if needed. That is the Fund needs to stand ready to absorb the debt position of the sovereign to prevent SFCs.

To ensure a robust design of the Fund, we consider that the sunspot remains a potential coordination device even in the presence of the Fund. As a result, there is always a risk of belief-driven sudden stops on the equilibrium path. However, as in Roch and Uhlig (2018), the Fund's intervention prevents that belief-driven sudden stops translate into SFCs.

In sum, during a fundamental-driven sudden stop, the Fund's intervention has to guarantee that the sovereign can honor its long-term liabilities given the negative spread. While, during a belief-driven sudden stop, the Fund's intervention needs to ensure that the sovereign can roll-over its maturing debt if needed. We say that such interventions are *minimal* as they correspond to the least possible interventions necessary to make sovereign debt safe. We therefore define the Fund's *minimal intervention policy* (MIP) in the following terms.

Definition 2 *(Minimal Intervention Policy (MIP)).* For a given state (θ, b_l), we say that the Fund implements a Minimal Intervention Policy (MIP) if $\bar{a}_l' = \underline{a}(\theta, b_l)$ where

I. If (18) binds, $\underline{a}(\theta, b_l) \in [\check{a}_l, \check{a}_l + \delta b_l]$ with $\check{a}_l \equiv \theta^- Z - \min\left\{\hat{a}_l'(\theta') : (18) \text{ binds} \wedge \pi(\theta'|\theta) > 0\right\}$.[19]
II. If (18) does not bind, $(s, \omega) \in \mathcal{C}(\varrho)$ and $\rho = 1$, then $\underline{a}(\theta, b_l) \in [\bar{\omega}_l - \delta b_l, \bar{\omega}_l]$.
III. Otherwise, $\underline{a}(\theta, b_l) = 0$.

We can now characterize the Fund's intervention in terms of default insurance. We start with a proposition stating that in the economy with the Fund, there is neither fundamental-driven nor belief-driven defaults. Proofs can be found in Appendix B.

Proposition 1 *(No Default). With the Fund's intervention, the sovereign does not default.*

As we have just seen, the Fund stands ready to provide the necessary resource in case of failed private debt auction. This means that there is no space for SFCs as the Fund has the capacity to perfectly substitute private lending in case of a roll-over crisis in the private bond market. A direct corollary of Proposition 1 is therefore the following.

Corollary 1 *(Safe Zone). With the Fund's intervention, the sovereign remains in the safe zone.*

In sum, the Fund's intervention is capable of preventing the occurrence of both fundamental and belief-driven debt crises. As a result, the economy always remains in the safe zone. That is, there is no multiplicity of equilibria anymore.

A broader corollary is that the results of Liu et al. (2022) on existence and uniqueness of a RCE generalize to our environment where, without the Fund, there can be SFCs (i.e. their Propositions 4 and 5 and Corollary 3).[20] In the environment of Liu et al. (2022) (i.e. without SFCs), the source of multiplicity comes from the fact that private lenders can always coordinate on autarky, when a non-autarkic equilibrium is feasible. The authors show that the Fund prevents such issue. In our environment, there is an additional source of multiplicity which owes to the coordination of beliefs. As we have just seen, the Fund's MIP prevents such multiplicity as the economy always remains in the safe zone. As a result, the Fund implements the unique constrained efficient allocation which features no autarky, no over-lending and no default – including SFCs.

6. Optimal maturity structure

In the context of self-fulfilling debt crises, the maturity structure of sovereign debt is crucial. Our framework unveils a novel tradeoff in the choice of maturity.

On the one hand, as noted by Cole and Kehoe (2000), long maturities reduce the amount of debt that needs to be rolled over every period and therefore shrink the crisis zone. On the other hand, shorter maturities ease the intervention of the Fund. This second component is new and is due to the Fund's MIP presented in Section 5. From Definition 2 Part I, the Fund's debt absorption can go up to $\check{a}_l + \delta b_l$ when (18) binds. Hence, long maturities (i.e. $\delta \to 1$) tighten the Fund's constraint as b_l appears on the right-hand side of (8).

[19] Alternatively, *MIP I* could be limited to the Fund's holdings of private debt and the commitment to implement the risk-sharing part of the Fund's contract; in this case, $\check{a}_l \equiv \theta^- Z$.

[20] Their Existence and First Welfare Theorems require interiority assumptions that are easily satisfied in our economies.

There is therefore a clear tradeoff in the choice of maturity. To see this, first define the gross financing needs (GFN) as

$$GFN(\delta; s, \omega, \bar{\omega}') = q_f(s, \omega, \bar{\omega}')(\bar{\omega}'_l - \delta\omega_l),$$

where $\bar{\omega}'_l = b'_l + \bar{a}'_l$. The GFN represents the total amount of debt that needs to be issued every period. The closer is δ to 1, the lower is the GFN. Conversely, the closer is δ to 0, the larger is the amount of debt to be issued every period. In that logic, the GFN provides a proxy for the risk of a roll-over crisis.

However, this argument holds only if the sunspot remains a potential coordination device in the presence of the Fund – as we assume. Otherwise, the optimal maturity is the one that minimizes Fund's private debt absorption, i.e. $\delta = 0$.

The Fund's MIP dictates that the Fund should absorb the least debt. This does not mean that the Fund's holdings of sovereign debt are always zero. As stated in Definition 2 Part I, the Fund's debt absorption can go up to $\breve{a}_l + \delta b_l$ when (18) binds. The Fund's holdings of private debt may therefore amount to δb_l. Such absorption ensures that the sovereign can honor its long-term liabilities in the presence of a negative spread.[21] Hence, the argument is the opposite of the one in the previous paragraph: the closer δ to 0 (1), the lower (larger) the amount of debt to be absorbed by the Fund every period.

Again, this argument holds only if (18) binds in the long run. If fundamental-driven sudden stops never realize in steady state, then the Fund does not need to activate its guarantee on long-term debt. There is therefore no reason to shorten maturity and the optimal maturity is the one that minimizes the risk of SFCs, i.e. $\delta = 1$.[22]

In contrast, if (18) binds in the long run, the optimal maturity is then the level of δ for which the maximum private debt absorption δb_l equates the maximum GFN. In other words, the optimal maturity is the one that balances the Fund's MIP with the risk of SFCs. Formally, define the *minimal absorption capacity* for a Fund contract with maturity δ as:

$$A^c(\delta) = \max\{GFN(\delta), \delta b_l\}. \tag{20}$$

We then denote the *optimal maturity* to be $\delta^* = \arg\min_{\delta\in[0,1]} A^c(\delta)$ and the *Required Fund Capacity* (RFC) for a Fund contract to be $A^c(\delta^*)$. Such maturity δ^* exists as the maximum GFN is decreasing in δ, while the opposite is true for the Fund's maximum private debt absorption.

Fig. 6 in Section 8 shows a graphical solution for the optimal maturity δ^* under the Fund's MIP when the private debt needs to be absorbed by the Fund. Given the above definition, the *minimal absorption capacity*, $A^c(\delta)$, corresponds to the upper contour of the two curves in the figure. In section 8, we quantify this optimal maturity for the case of Italy.

7. Central Bank intervention with a constrained Fund

In this section, we introduce the Central Bank (CB) as a way to complement the Fund's intervention described in Section 5. First, we briefly discuss the *rationale* for constraining the CB's asset purchases to risk-free assets; more precisely, to assets without *fundamental* risk, and to absorb at most *belief-driven* risks, which can vanish with its purchases. We argue that the Fund, with its contract design, can better discriminate between fundamental and belief-driven risks. In particular, the Fund can signal if a CB sovereign debt purchase intervention is free from (fundamental) default and when it should not take place because the sovereign debt can become unsustainable. Second, we discuss when the CB intervention is needed due to the limited absorbing capacity of the Fund.

7.1. A constraint on central bank asset purchases

In executing (sovereign) asset purchase programs, CBs purchase domestic government bonds on the secondary market with interest-bearing reserves. Abstracting from money (cashless economy), private asset purchases, and standard repo operations, we can simplify the budget constraint of a CB as:

$$q_p(s, b, b')\left(b'_l - \delta b_l\right) - \mathcal{R}' + \mathcal{T} = (1 - \delta + \delta\kappa)b_l - (1 + r)\mathcal{R},$$

where $q_p(s, b, b')$ is the bond price in the secondary market as in equation (4). Specifically, the CB buys long term government bonds from private lenders b_l with reserves $\mathcal{R}$, on which it pays an interest rate r, which we assume is risk-free. $\mathcal{T}$ are transfers between the CB and the government or, when it exists, the Treasury.

To simplify the exposition of the argument, let's assume that $\kappa = \frac{r}{1+r}$, so that from equation (4) the risk free price, $q_p = \frac{1-\delta+\delta\kappa}{1+r-\delta} = \frac{1}{1+r}$. By imposing that reserves are safe in the sense that they are fully backed by the asset, i.e. $\mathcal{R} = \frac{b_l}{1+r}$, then $\mathcal{T} = 0$. If instead $q_p(s, b, b') < \frac{1}{1+r}$ from equation (4) it must be that the asset carries credit risk so that a positive spread compensates for the probability of default in the future.

What happens when the country defaults? In the event of default the Treasury needs to recapitalize the CB to guarantee that reserves are safe. Recapitalization can be on the spot, in which case the CB will never have negative capital, or gradual, in which case the CB will operate with negative capital for some time. Let's make the assumption that in case of default the

[21] See Footnote 19.

[22] As shown by Liu et al. (2022), in the absence of both fundamental and belief-driven sudden stops in steady state, the maturity is irrelevant.

government reneges the entire amount of debt and loses market access thereafter, i.e. $b_l = b_l' = 0$. The government budget constraint of the CB then becomes

$$-\mathcal{R}' + (1+r)\mathcal{R} = -\mathcal{T},$$

where the nominal amount of reserves at time t is equal to the defaulted bonds, i.e. $\mathcal{R} = \frac{b_l}{1+r}$. Let's consider the two extremes: recapitalization on the spot and recapitalization by means of a consol payment. In case of recapitalization on the spot, the Treasury needs to have enough deposits to cover the entire amount of unfunded liabilities. In that case, $\mathcal{R}' = 0$ and $\mathcal{T} = -b_l$. In the case of a consol payment, the value of reserves is guaranteed by future payments by the Treasury. Reserves will remain constant, i.e. $\mathcal{R}' = \frac{b}{1+r}$. This will require transfers from the government to the CB $\mathcal{T} = -\frac{r}{1+r} b_l$. In either case, to guarantee that reserves are safe, the losses of the CB need to be covered by the Treasury of the defaulting government, this means that the CB has *de facto* full seniority over private lenders.

In the case of the US, any credit risk taken by the Federal Reserve Board (FRB) needs to be backed by the US Treasury and an agreement had to be reached upfront between the US Treasury and the FRB before starting the credit easing programme. In Europe, absent a Euro-Area Treasury, sovereign bond purchases are delegated to the national CBs with the back of national treasuries. Therefore, either purchases do not carry credit risk, or it must be that domestic CBs have implicitly full seniority over their sovereign debt, something that the ECB has explicitly excluded. As shown by Bassetto and Caracciolo (2021), the alternative in case of default would be for domestic CBs to permanently roll over Target-2 liabilities vis a vis other Eurosystem CBs. But this would eventually result in debt mutualization as in Mackowiak and Schmidt (2022).

Within the framework in which the ECB operates, asset purchases cannot carry default risk. Either sovereign bonds are safe, or the ECB intervention makes them safe by preventing the occurrence of SFCs. Indeed, with both the Outright Monetary Transaction (OMT) and the Transmission Protection Instrument (TPI), the ECB wanted to equip itself with instruments specifically designed to address SFCs, drawing a clear separation between liquidity and solvency problems. As a precondition, the TPI can only be activated when debt is sustainable. As shown by Bianchi and Mondragon (2022), it is precisely when sustainability is at risk that SFCs are most likely to arise. Therefore, the debt sustainability issue must be addressed to activate TPI or similar ECB programmes.

The two instruments differ substantially in how the DSA is conducted. In the OMT, ECB's interventions go hand-in-hand with an ESM program, establishing a contractual relationship with the sovereign similar to that of the Fund (although without the cyclical insurance component). The TPI, however, relies on an initial DSA assessment but does not include a contractual relation over time.[23]

The alternative presented here is closer to the OMT setting. Also the TPI, however, if jointly activated with a well-designed stability Fund, can deactivate any possible SFCs and provide adequate insurance to markets, provided that together they cover the RFC, the fact that the CB stands ready to activate TPI deactivates any possible SFCs.[24]

Finally, it should be noted, following the argument presented above, that with the Fund, when the debt sustainability constraint is binding a negative spread arises – as explained in Section 5, so that $q_p(s, b, b') > \frac{1}{1+r}$, and the purchase of new bonds will require $\mathcal{T} < 0$. That is, the government will have to offset CB losses and, therefore the CB should stop purchasing the sovereign debt. In other words, the Fund not only (perfectly) complements the CB in the purchase of sovereign debt – for example, the ECB with TPI – but also complements it, by providing the right signal of when the CB should stop purchasing sovereign debt.

7.2. Central bank interventions as prudential-robust policies

Given that a Fund contract already provides insurance to normal fluctuations, the RFC is high, when a relatively high level of debt is needed to safely sustain the consumption plan of a debtor country. Nevertheless, if the country is small the Fund may have this capacity, but possibly not with a relatively large (and impatient) country. In this case, additional capacity to absorb debt is needed to guarantee that the sovereign debt is safe, even if in equilibrium such capacity may not be needed. It is in this context where the CB can credibly provide such capacity and both institutions act together as LOLR, with the CB not absorbing (fundamental) risk. However, such CB capacity to intervene (e.g. of the ECB to activate the TPI programme) may only be needed rarely or uncorrelated with other countries with Fund contracts, in both cases the demands on the CB can be easier to be managed. In particular, as we have seen, GFN will not need to be absorbed when the CB intervention gives credibility to the announcements of the Fund. But short of full credibility, some absorption may be needed to be a *robust* LOLR, and the RFC accounts for this. These are risks that require quantitative estimation, which we explore in the next section. The theory also predicts that there are scenarios for which the joint intervention of the Fund and the CB may not be enough to cover the RFC. In such cases, there is a need for additional coordination with a Treasury with the capacity to commit additional current or future revenues (i.e. primary surpluses).

[23] See Bernoth et al. (2022) for details on the comparison between TPI and OMT.

[24] The ESM complements well the ECB purchases in addressing SFC concerns also given its capacity to perform primary purchases directly at the auctions, something that the ECB cannot do.

8. Quantitative analysis

In this section, we calibrate the model to Italy, assess the fit of the model to the data and compute welfare gains. We then conduct a series of steady-state analyses before deriving the optimal maturity and assessing the debt absorption of the Fund.

8.1. Calibration

We calibrate the parameters by fitting the sovereign debt model without the Fund presented in Section 4 to Italy over the period 1992 to 2019.[25] Unlike Liu et al. (2022), the frequency of the model is yearly.

Similar to Liu et al. (2022), we calibrate the productivity growth rate shock γ_t with a Markov regime-switching AR(1) process to the sample productivity series of Italy. We choose a specification of 2 regimes that we denote by $\varsigma \in \{1, 2\}$, with the first regime capturing the crisis period (i.e. the Great Financial Crisis) observed in the data. Specifically, we estimate the following model for the (net) growth rate $\gamma_t - 1$ with the expectation maximization (EM) algorithm of Hamilton (1990):

$$\gamma_t - 1 = (1 - \rho(\varsigma_t))\mu(\varsigma_t) + \rho(\varsigma_t)(\gamma_{t-1} - 1) + \sigma(\varsigma_t)\epsilon_t, \tag{21}$$

where ς_t denotes the regime at t, $\rho(\varsigma_t)$, $\mu(\varsigma_t)$, $\sigma(\varsigma_t)$ are the regime-specific autocorrelation, mean and variance of the process, respectively, and ϵ_t follows an i.i.d. standard normal distribution. The parameter values are reported in Table 1. We further discretize the shock process using the method of Liu (2017) with 5 grid points for each regime. This gives a total of 10 growth states γ.

Table 1
Parameters of the regime switching process.

	$\mu(\varsigma)$	$\rho(\varsigma)$	$\sigma(\varsigma)$	Π	$\varsigma' = 1$	$\varsigma' = 2$	invariant dist.
$\varsigma = 1$	−0.0336	0.9018	0.0009	$\varsigma = 1$	0.6633	0.3367	0.0372
$\varsigma = 2$	0.0009	0.2167	0.0020	$\varsigma = 2$	0.0130	0.9870	0.9628

Note: ς denotes the current regime of growth shock, and ς' denotes that of the next period. We consider two regimes, $\varsigma \in \{1, 2\}$, with transition matrix Π. $\varsigma = 1$ captures the period of the Great Financial Crisis. The regime-specific autocorrelation, mean and variance of the process are denoted by $\rho(\varsigma_t)$, $\mu(\varsigma_t)$, and $\sigma(\varsigma_t)$, respectively.

We detrend the variables representing allocations – except for labor n where we normalize the time endowment to 1 – by dividing them by θ_{t-1}. For instance, we denote by $\tilde{c}_t$ the detrended form of c_t such that $\tilde{c}_t = \frac{c_t}{\theta_{t-1}}$ represents the deviation from the trend.

Table 2 summarizes the value of each parameter in the model. The preference parameters for labor supply are set to $\xi = 1.275$ and $\zeta = 0.2$ to match the average fraction of working hours and its volatility relative to output, respectively. The risk-free interest rate is set to $r = 1.32\%$, the average real short-term interest rate of the Euro area. We further set $\delta = 0.839$ and $\kappa = 0.2172$ to match the average Italian bond maturity and coupon rate (coupon payment to debt ratio), respectively. Finally, we fix $\beta = 0.97$ to match the average indebtedness relative to annual output. The production function is Cobb-Douglas $f(n) = n^{\alpha}$, and we set $\alpha = 0.5295$ to match the average labor share in Italy.

Table 2
Parameter values.

Parameter	Value	Definition	Targeted Moment
A. Direct measures from data			
α	0.5295	labor share	labor share
r	0.0132	risk-free rate	annual real short-term rate
δ	0.839	bond maturity	bond maturity
κ	0.2172	bond coupon rate	bond coupon rate
B. Based on model solution			
β	0.97	discount factor	average b/y
d_0	-0.297	productivity penalty	average spread
d_1	2.195	productivity penalty	corr$(\tau/y, y)$
ϱ	0.004	probability $\rho = 1$	corr(spread, y)
λ	0.054	return probability	$\sigma(\tau/y)/\sigma(y)$
ζ	0.2	labor elasticity	$\sigma(n)/\sigma(y)$
ξ	1.275	labor utility weight	average n
C. By assumption			
Z	0	Fund's outside option	

Note: The variable $\sigma(\cdot)$ denotes the volatility and τ/y denotes the primary surplus (i.e. $\theta f(n) - c$) over output.

[25] The calibration starts in 1992 due to data availability and ends in 2019 owing to the pandemic.

Table 3
Model outcome.

Variable	Data	SFC		No SFC	
		Without Fund	With Fund	Without Fund	With Fund
A. Targeted Moments					
$b'/y\%$	117.64	118.00	123.70	119.10	176.8
$n\%$	38.64	38.87	39.09	38.80	39.51
spread%	2.50	0.48	-0.04	0.13	-0.03
$\sigma(\tau/y)/\sigma(y)$	1.09	1.38	0.91	0.96	0.91
$\sigma(n)/\sigma(y)$	0.75	0.75	0.74	0.74	0.75
corr(spread, y)	-0.16	-0.29	-0.71	-0.37	-0.66
corr(τ/y, y)	0.29	0.42	0.97	0.54	0.98
B. Non-Targeted Moments					
σ(spread)	0.96	0.66	0.01	0.08	0.01
$\sigma(c)/\sigma(y)$	1.27	0.88	0.25	0.91	0.20
corr(c, y)	0.53	0.61	0.77	0.64	0.85
corr(n, y)	0.68	0.56	0.98	0.51	0.99

Note: The variable $\sigma(\cdot)$ denotes the volatility and τ/y denotes the primary surplus (i.e. $\theta f(n) - c$) over output.

We consider a quadratic default penalty as in Chatterjee and Eyigungor (2012). Formally, $\theta^D = \theta - \max\{0, d_0\theta + d_1\theta^2\}$ with $d_1 > 0$. We set $d_0 = -0.297$ and $d_1 = 2.195$ to match the average spread and the correlation between the primary surplus and output, respectively. Moreover, we fix $\lambda = 0.054$ to match the volatility of the primary surplus relative to output as in Arellano (2008). Finally, regarding the sunspot, we set $\varrho = 0.004$ to match the correlation between the spread and output. Such value is consistent with the estimate of Bocola and Dovis (2019) for Italy and makes SFCs very rare events.

Note that similar to Bocola and Dovis (2019) and Bocola et al. (2019), our calibration differs from the standard calibration in the sovereign debt literature based on emerging economies. We adopt a rate of time preference $\beta = 0.97$ which is close to $\frac{1}{1+r} = 0.987$. Moreover, we target an average spread of around 2%, while this statistic is at least twice as large in emerging economies. Finally, we seek to match the counter-cyclicality of the Italian primary surplus which again diverges from the one observed in emerging economies. As a result, the "front-loading" motive is expected to be not too pronounced in the calibrated model. The sovereign should therefore use the private bond market in the economy without the Fund mostly for "consumption smoothing" purposes.

8.2. Model fit and comparison

We now assess the fit of the model with the data. We also compare the economy with and without the Fund. Finally, we compare the economy with and without SFC.

We start with the fit of the model to the data. As one can see in Table 3, the economy without the Fund and with SFC presented in Section 4 replicates well the targeted and non-targeted moments related to Italy. The indebtedness is substantial as it exceeds 115% of GDP. The correlation of the spread with respect to output is also very close to its data counterpart. In terms of primary surplus, the correlation is around 0.4 and the relative volatility is above 1. Moreover, the share of hours worked is less volatile than output and is around 38% as in the data. However, the average spread is below 1%, while it is above 2% in the data. The average spread in the model is nevertheless close to the one targeted by Bocola and Dovis (2019).[26]

In terms of non-targeted moments, the model generates a realistic volatility of spread. It also produces correlations of consumption and labor relative to output close to the data. However, consumption is not enough volatile in the model.

Comparing the economy with and without the Fund, one notices three main elements. First, the level of private debt is larger in the economy with the Fund. Note that Table 3 only reports the share of private debt. For instance, the Fund's average holding of debt amounts to 12.2% of output with SFC and 5.4% without. Second, the economy with the Fund records an average negative spread due to the occurrence of fundamental sudden stops as explained in Section 5. Third, consumption is less volatile, while the primary surplus becomes highly counter-cyclical with the Fund. As we will show in the next section, the smoother consumption path and the greater debt absorption are the main sources of welfare gains of the Fund.

The basis of calibration relies on the economy developed by Cole and Kehoe (2000). We now compare this economy with the one without SFC (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008). We start with the economy without the Fund. As one can see, the average level of indebtedness in the economy with SFC is lower, while the average spread and its volatility are higher than in the economy without SFC. The larger and more volatile spread is due to the presence of the crisis zone coupled with the realization of $\rho = 1$ and makes debt accumulation more costly. Moreover, the spread correlates less negatively with output in the presence of SFC. Hence, even though SFCs are rare, they significantly affect the outcome of the model.

[26] The model can generate an average spread above 2% by lowering β. However, this causes corr(spread, y) to become positive.

Table 4
Welfare comparison at zero initial debt.

State		Welfare Gains (%)		Maximal Debt Absoption (% of GDP)			
		With Fund		With Fund		Without Fund	
		SFC	No SFC	SFC	No SFC	SFC	No SFC
$\rho = 0$	$\gamma = \gamma_{min}$	0.50	0.80	180	250	159	171
$\rho = 0$	$\gamma = \gamma_{med}$	0.16	0.42	144	194	136	141
$\rho = 0$	$\gamma = \gamma_{max}$	0.01	0.38	126	168	112	113
$\rho = 1$	$\gamma = \gamma_{min}$	0.50	-	180	-	158	-
$\rho = 1$	$\gamma = \gamma_{med}$	0.16	-	144	-	136	-
$\rho = 1$	$\gamma = \gamma_{max}$	0.01	-	126	-	112	-
Average		0.11	0.41				

Note: The table reports welfare gains of the Fund's intervention at zero initial debt in consumption equivalent terms. Given the functional form of the utility function, welfare gains for a specific s correspond to $\exp\left[(W^b(s,0,0) - V(s,0))(1-\beta)\right] - 1$ where W^b and V are the values of the sovereign in the economy with and without the Fund, respectively. The case without SFC is the one in Liu et al. (2022).

Regarding the Fund with and without SFC, there are two main differences.[27] First, without SFC, the Fund enables the sovereign to accumulate more private debt, and, second, the negative spread is less pronounced on average. However, in terms of business cycle dynamics, the two economies are very much alike.

8.3. Welfare analysis

The previous subsection compared the economy with and without the Fund in terms of targeted and non-targeted moments. We now assess the gains of accessing the Fund in terms of welfare and gauge the benefits of insuring against SFCs.

Table 4 presents the welfare gains of the Fund's intervention in consumption equivalent terms. We compute such gains for the economy with the Fund relative to the economy without the Fund at zero initial debt holdings. More precisely, given the functional form of the utility function, welfare gains for a specific s correspond to $\exp\left[(W^b(s,0,0) - V(s,0))(1-\beta)\right] - 1$ where W^b and V are the values of the sovereign in the economy with and without the Fund, respectively.[28]

Welfare gains are significant with the Fund's intervention, especially in the absence of SFCs. With zero initial debt, the consumption-equivalent welfare gains are on average 0.1% with SFC and 0.4% without. Moreover, the largest welfare gains are recorded in low growth states. Thus, the Fund's intervention is mostly valued when the sovereign is in a difficult economic situation.

Comparing the two specifications, one clearly sees that the presence of SFCs drastically impacts the sovereign's welfare. There is a trade-off behind this result. On the one hand, one would expect that stabilizing the debt in an economy with higher spreads (see Table 3) would result in higher welfare gains but, on the other hand, the economy with SFCs is likely to have lower outside values – particularly, in low growth states – for both the borrower and the lender, which makes the Fund intervention less effective in terms of welfare. In our simulated economy, in which we do not account for some of the costs of having high spreads (e.g. in the private credit market or in the transmission of monetary policy), the latter effect dominates and the welfare gains are lower in the Fund with SFC.[29]

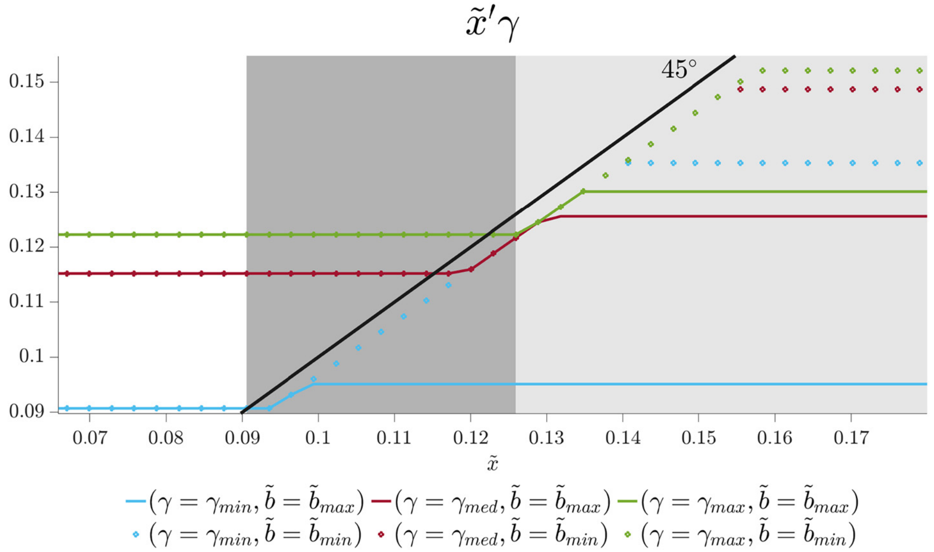
The last part of Table 4 depicts the maximal debt absorption capacity of each economy. Consistent with the results in Table 3, the Fund always allows the sovereign to accumulate more debt. Moreover, consistent with what has been said previously, the Fund without SFC enables a greater debt accumulation than the Fund with SFC.[30]

8.4. Steady state analysis

As explained in Section 5 and detailed in Appendix A, the recursive formulation of the Fund relies on the relative Pareto weight x. This variable is key to the dynamic of the model economy as it represents a sufficient statistic of the contract's binding constraints. We first explain the dynamic of the relative Pareto weight before simulating the economy in steady state.

Fig. 1 displays the law of motion of the relative Pareto weight. The dark grey region represents the ergodic set which defines the steady state of the economy. The light grey region represents the basin of attraction of the ergodic set.[31] As one

[27] The Fund without SFC is the one exposed by Liu et al. (2022).
[28] The details of the welfare computation can be found in Ábrahám et al. (2022) and Liu et al. (2022).
[29] Note that we record lower welfare gains than Liu et al. (2022) as we adopt a larger β and a different specification for the default penalty.
[30] The lower debt absorption capacity can be related to the fact that in the Fund without SFC, the no-over-lending constraint binds less frequently. The probability of such constraint binding in steady state is twice as large in the Fund with SFC compared to the Fund without SFC.
[31] Appendix C shows the evolution of the relative Pareto weights in the economy without SFC. A close comparison of both ergodic sets shows the subtle differences of their bounds that translate into the welfare differences of Table 4.



Note: The figure depicts the law of motion of the detrended relative Pareto weight for different growth states and different private debt levels. The y-axis corresponds to the prospective relative Pareto weight, while the x-axis represents the current weight. The dark grey x-axis region is the ergodic set which defines the steady state of the economy. The light grey and white regions are the basin of attraction of the ergodic set.

Fig. 1. Evolution of the relative Pareto weight in steady state as a function of $(\gamma, \tilde{b}, \tilde{x})$.

can see, the convergence path to the steady state depends on the level of privately held debt. Especially, the larger is the level of private debt, the closer the economy gets to the ergodic set. Moreover, the Fund's constraint is binding in steady state. This means that the optimal maturity is the one for which the GFN equates $\delta\tilde{b}_l$ as exposed in Section 6.

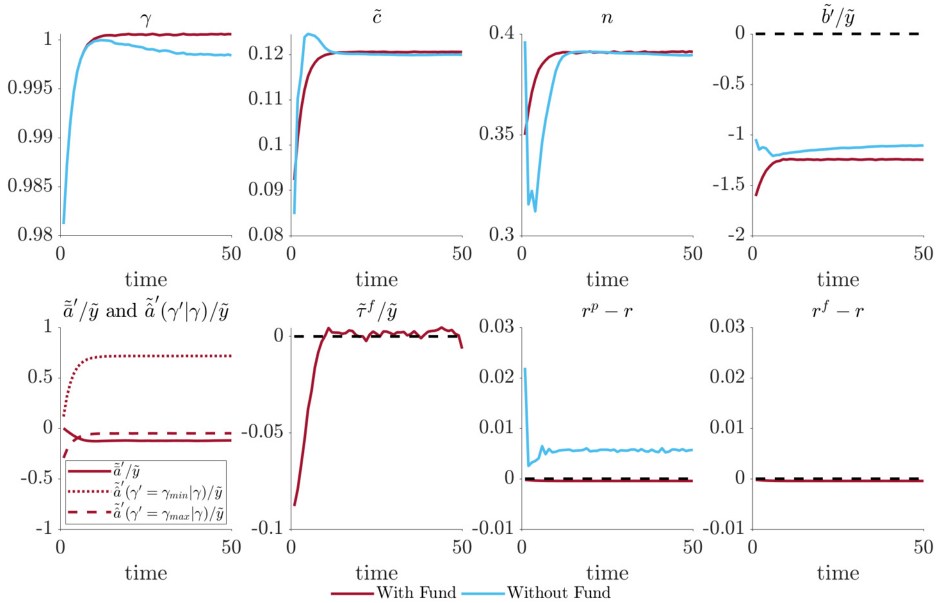
To get an idea on the Fund's intervention, we conduct two exercises. In the first one, we analyze the impact of a stark negative growth shock on the economy. In the second, we perform a *robustness* exercise, by adding a roll-over crisis on top of the sudden drop in growth and allowing private lenders to misbelieve the Fund announcement. We present both impulse response functions and simulation plots.

To generate impulse response functions, we simulate 5,000 independent shock histories for 50 periods starting with the lowest growth shock as well as initial debt holding and relative Pareto weight drawn from the ergodic set. To gauge the impact of the Fund's intervention in this exercise, we simulate both the economy with and without the Fund in parallel. Note that we select the initial shock and the initial debt such that the economy without the Fund is in the crisis zone.

Fig. 2 depicts the impulse response functions resulting from a stark negative growth shock on selected key variables. The economy without (with) the Fund is in blue (red). In the very first periods following the negative shock's realization, the sovereign increases its indebtedness in both cases with and without the Fund. Importantly, there is no default on impact in the economy without the Fund as we are in the crisis zone and the private lenders expect repayment (i.e. $\rho = 0$). Nonetheless, debt accumulation is limited in the economy without the Fund by the large positive spread. Consistent with Definition 2, the Fund only provides Arrow securities and lets the private bond market absorb the debt on impact. Note that the wedge in γ between the two economies is due to the occurrence of defaults along the simulated path in some economies without the Fund and the underlying default penalty.

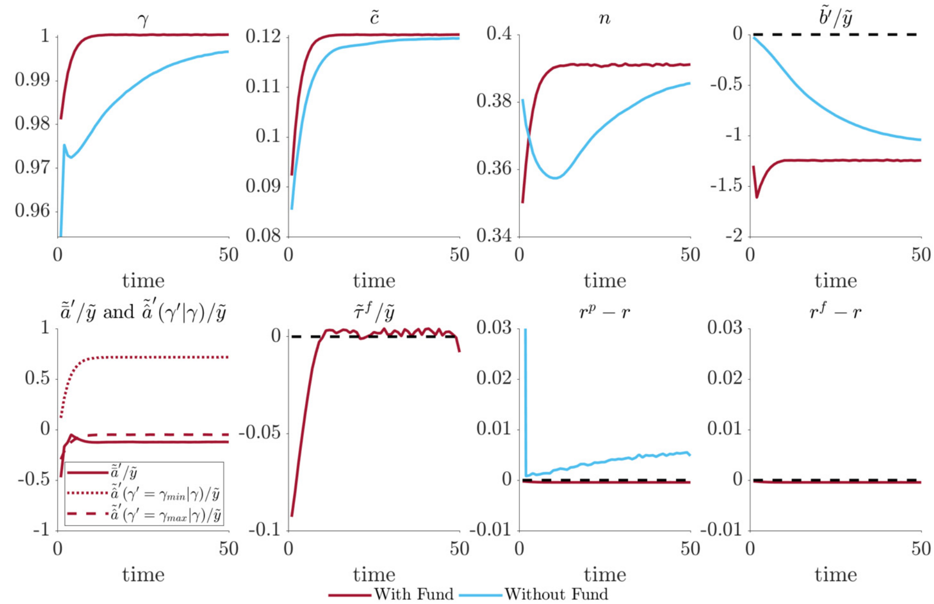
Fig. 3 depicts the impulse response functions resulting from a stark negative growth shock coupled with a SFC (i.e. $\rho = 1$ in the crisis zone). Again, the wedge in γ between the two economies is due to the occurrence of defaults in some economies without the Fund and the underlying default penalty. On impact, the economy without the Fund defaults as the private lenders, expecting a default, coordinate not to lend to the sovereign. Regarding the economy with the Fund, on impact, the level of private debt is lower than the one depicted in Fig. 2 due to the rollover crisis. This is exactly when the Fund intervenes absorbing GFN, since the sunspot remains a coordination device given private lenders' misbeliefs. Particularly, it reacts to the failed auction with additional insurance and, most importantly, debt. This prevents debt repudiations and the underlying large increase in spread observed in the economy without the Fund. After the sunspot realization, the sovereign largely increases its indebtedness in the private bond market and reduces its holdings of debt in the Fund.

Besides impulse response functions, we also show the simulation of a steady state path with and without SFC. Fig. 4 depicts the latter. Again, the economy without (with) the Fund is in blue (red). The grey area represents the region in which the economy without the Fund defaults. One can see that defaults occur after a sharp reduction in growth. The Fund prevents the occurrence of such an event by providing more insurance. As one can see, the transfer largely shifts at the outbreak of default to sustain the level of outstanding debt. However, the Fund does not absorb any debt at the default's outbreak given Definition 2 part III. Having said that, the Fund does absorb debt when the no-over-lending constraint binds given Definition 2 part I.



Note: The figure depicts impulse response functions following a negative growth shock without SFC on selected key variables. The economy without (with) the Fund is in blue (red). Impulse response functions are obtained by averaging the simulation of 5,000 independent shock histories for 50 periods starting with $\gamma = \gamma_{min}$, $\rho = 0$ and initial debt holding and relative Pareto weight drawn from the ergodic set.

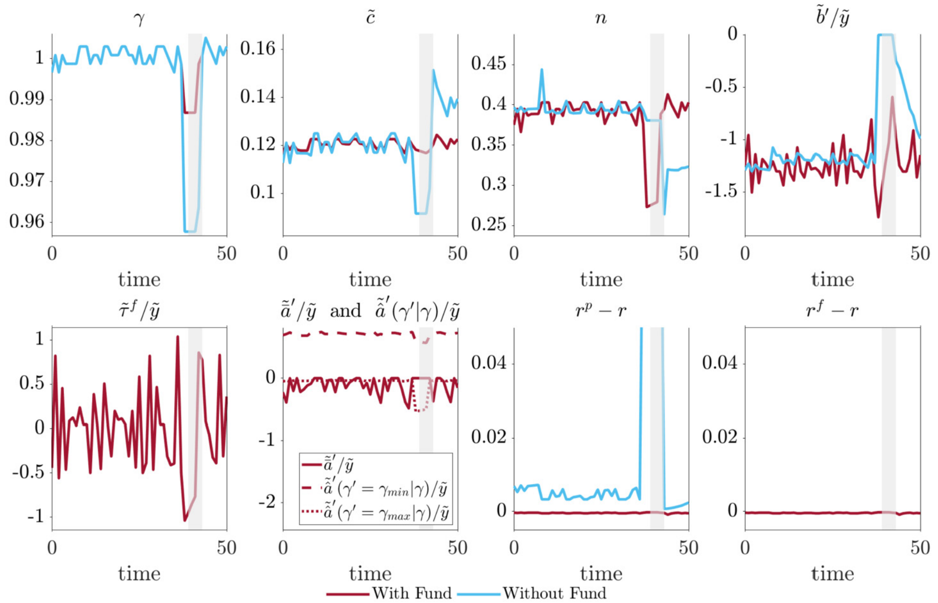
Fig. 2. Impulse response functions to a negative γ shock without SFC. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)



Note: The figure depicts impulse response functions following a negative growth shock with SFC on selected key variables. The economy without (with) the Fund is in blue (red). Impulse response functions are obtained by averaging the simulation of 5,000 independent shock histories for 50 periods starting with $\gamma = \gamma_{min}$, $\rho = 1$ and initial debt holding and relative Pareto weight drawn from the ergodic set. We assume that private lenders coordinate on the sunspot ρ.

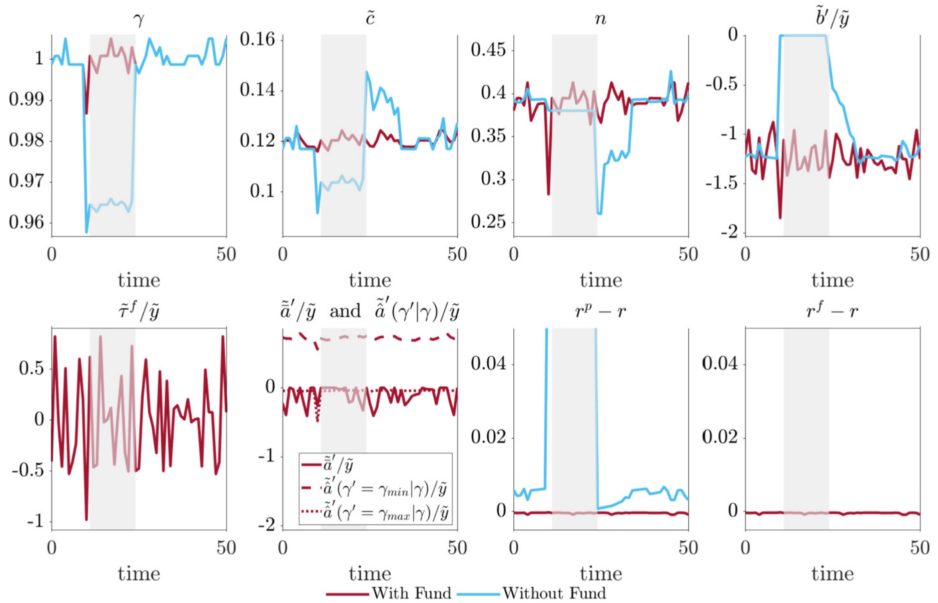
Fig. 3. Impulse response functions to a negative γ shock with SFC and LOLR absorption.

Fig. 5 depicts a steady state path with SFC. The main difference with the dynamic presented in Fig. 4 is that the Fund absorbs some debt at the default's outbreak. Such absorption is relatively important (i.e. around 50% of output) and is necessary to prevent the occurrence of default. Note however that the Fund's debt absorption in the case of binding no-over-lending limit can be even larger. We come back to the size of the intervention when we discuss the intervention of the CB.



Note: The figure depicts the simulation of a specific steady state path without SFC on selected key variables. The economy without (with) the Fund is in blue (red). The grey area represents the region in which the economy without the Fund defaults.

Fig. 4. Simulation of a steady state path without SFC.



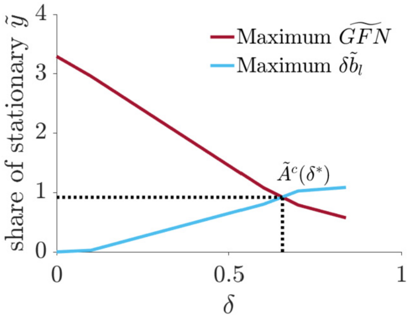
Note: The figure depicts the simulation of a specific steady state path with SFC on selected key variables. The economy without (with) the Fund is in blue (red). The grey area represents the region in which the economy without the Fund defaults. We assume that private lenders can coordinate on the sunspot ρ.

Fig. 5. Simulation of a steady state path with SFC and LOLR absorption.

8.5. Maturity analysis

Following Section 6 and our calibration, we are interested in the optimal maturity structure of Italy under the Fund's intervention.

As shown in Fig. 1, with the existing debt maturity of Italian debt, the Fund's no-over-lending constraint binds in steady state. As a result, the optimal maturity is the level of δ for which the maximum GFN equates to the maximum private debt



Note: The figure depicts the graphical solution for the optimal maturity δ^* under the Fund's MIP when the private debt needs to be absorbed by the Fund. The red line corresponds to the maximum GFN and the blue line to the maximum $\delta\tilde{b}_l$ both as a share of stationary output. The minimal capacity absoprtion, $\tilde{A}^c(\delta)$, represents to the upper contour of the two curves, while the optimal maturity δ^* is the crossing of the two curves.

Fig. 6. Optimal maturity.

absorption, $\delta\tilde{b}_l$. Fig. 6 depicts the optimal maturity emerging from this tradeoff using the calibration for Italy. To obtain this figure, we first re-compute the economy without the Fund for different values of δ and then re-compute the Fund with the underlying new δ and outside option.

One can see that the maximum GFN is decreasing in δ, while the opposite is true for the Fund's maximum private debt absorption. Moreover, both objects seem to be a non-linear function of δ. In the case of Italy, for $\varrho = 0.004$, we find an optimal δ^* around 0.65 which means an average maturity of roughly 2.9 years.

The average maturity on Italian bonds is around 6.2 years in the period considered. Thus, an optimal maturity of 2.9 years is more than half shorter on average compared to the data. This wedge between the optimal and the factual maturity is relatively large given that the probability of a SFC is relatively low in our calibration.

Besides the maturity, Fig. 6 suggests a RFC, $\tilde{A}^c(\delta^*)$, of the order of 90% of stationary GDP. This figure is below the needed absorption capacity of 105% under the current average Italian maturity. Nevertheless, it remains well above what the ESM could reasonably absorb for a single country. In light of our discussion in Section 7, a complementary intervention of the ECB seems necessary here. Such intervention would be feasible under the *Transmission Protection Mechanism* (TPI). We further develop this argument in the next subsection.

8.6. Debt absorption and the need for central bank intervention

As seen previously, in the presence of SFCs, the Fund might need to be able to absorb a substantial amount of debt. We now quantify the intervention of the central bank in the case in which the Fund might be financially constrained.

We simulate 5,000 independent growth shock histories for 500 periods and discard the first 100 periods to make sure the initial conditions do not matter. This enables us to compute the distribution of GFN and the Fund's absorption of private debt in steady state.

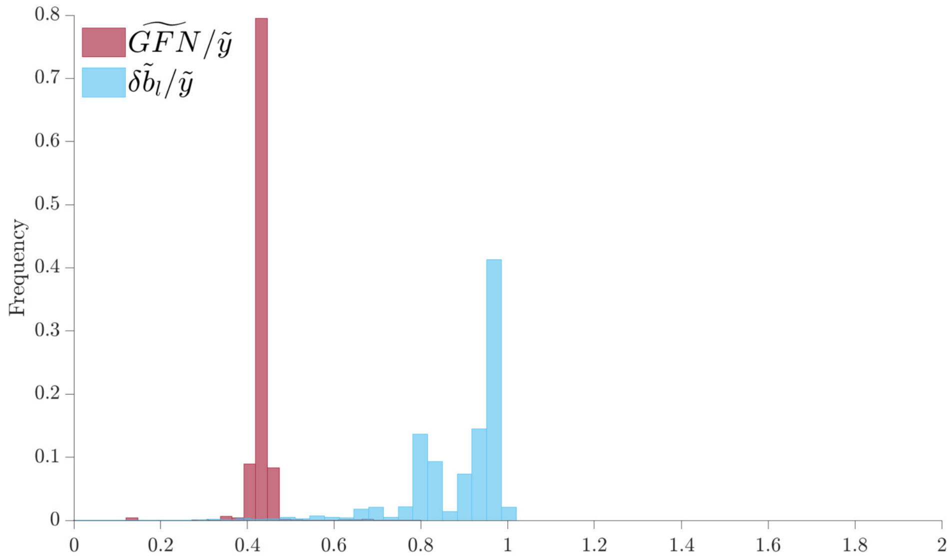
Fig. 7 presents the histogram of GFN and $\delta\tilde{b}_l$ relative to GDP in steady state for the Italian δ. Regarding the GFN, the greatest part of the mass lies around 50% of GDP. Moreover, one can see that the distribution has long tails. While on some rare occasions, the GFN corresponds to roughly 10% of Italian GDP, it may also happen – albeit even more rarely – that the Fund would have to absorb 8 times that amount. Regarding the private debt absorption of the Fund, we see that the largest part of the mass lies around 100% of Italian GDP. The left tail is nonetheless larger than the GFN.

Given that the Fund is unlikely to be financially capable of absorbing such levels of debt, this gives room for the CB's intervention. If the Fund can only take up to say 10% of GDP, the CB would need to be able to absorb the remaining part. That is, in terms of $\delta\tilde{b}_l$, the CB would have to be able to absorb around 90% of the country's GDP in debt. In terms of GFN, this figure amounts to 40% on average. Nevertheless, the probability of such an event is very small, since as we have noted at the beginning of this section, in our calibration, the probability of drawing $\rho = 1$ is 0.004. In contrast, the probability that at the steady-state the Fund's no-over-lending constraint is binding is nonetheless larger, i.e. around 0.06.

Given the previous section, we can also compute the distribution of the GFN and $\delta\tilde{b}_l$ for the optimal maturity δ^*. As one can see in Fig. 8, the mass related to $\delta^*\tilde{b}_l$ (GFN) largely shifts to left (right) compared to Fig. 7. In other words, the GFN and the private debt absorption come closer to each other. Furthermore, the mass related to Fund's private debt absorption, $\delta^*\tilde{b}_l$, lies around 80% of the country's GDP in debt. As a result, reducing the current average Italian maturity would be beneficial as this would reduce the maximum RFC.

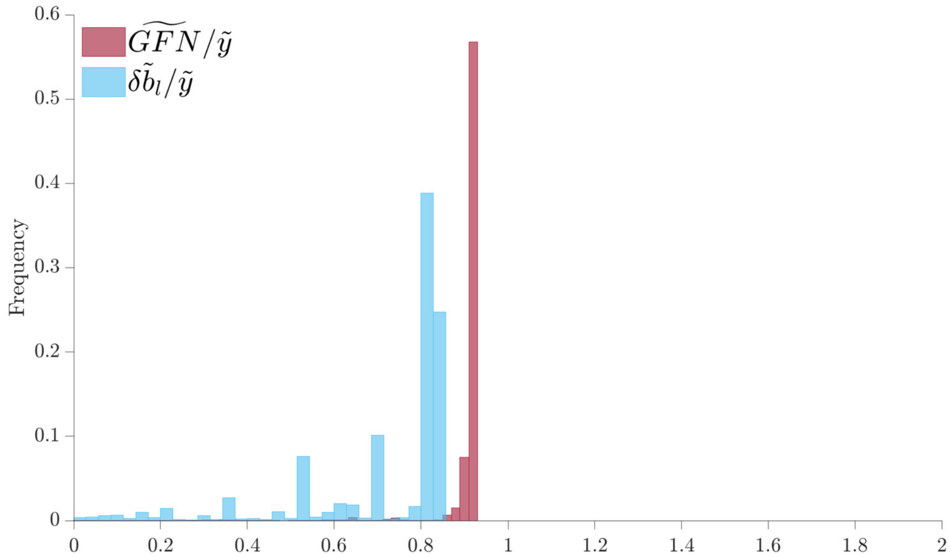
9. Conclusion

A well-designed *lender of last resort*, LOLR, "should minimize its lending intervention while addressing existing market failures" (as Thomas Cooley would have said). In the sovereign debt market, these market failures range from excessive precautionary savings, when borrowing in bad times is excessively costly, to all possible forms of costly defaults, current or



Note: The figure depicts the distribution of GFN and the Fund's absorption of private debt in steady state for the Italian maturity, i.e. $\delta = 0.839$. The red bars correspond to the GFN and the blue bars to $\delta\tilde{b}_l$ both as a share of output. The distribution is obtained by the simulation of 5,000 independent growth shock histories for 500 periods where the first 100 periods are discarded.

Fig. 7. Absorption at Italian δ.



Note: The figure depicts the distribution of GFN and the Fund's absorption of private debt in steady state for the optimal maturity, i.e. $\delta = 0.65$. The red bars correspond to the GFN and the blue bars to $\delta\tilde{b}_l$ both as a share of output. The distribution is obtained by the simulation of 5,000 independent growth shock histories for 500 periods where the first 100 periods are discarded.

Fig. 8. Absorption at optimal δ.

future (outright or partial default, inflation if debt is nominal, dilution if debt is long-term), which also includes excessive lending, since it reflects that future revenues (i.e. primary surpluses) cannot cover the full cost of current debt liabilities, as well as non-fundamental risks purely driven by expectations resulting in self-fulfilling crises (SFCs) with positive probability. All of them are market failures because neither the sovereign nor private lenders internalize the associate social costs (e.g. positive spreads cover the expected losses of private lenders, but not the deadweight losses of default, and often create additional market distortions; e.g. for the transmission of monetary policy).

On July 26, 2012, Mario Draghi, made his famous "whatever it takes" speech,[32] and the markets reacted to it with plunging spreads, that were too high for 'stressed' states' debt to be sustainable. It was not just rhetoric, in the back there was additional institutional backup (the just created European Stability Mechanism (ESM)) and the European Central Bank (ECB) was ready to intervene for the first time in the sovereign debt market with its Outright Monetary Transactions (OMT) programme. In the end, the combination of these elements made the intervention unnecessary, as if an 'effective lender of last resort', had dissipated an SFC, with the ECB dissipating a belief crisis and the ESM taking care of more structural debt sustainability problems, each institution acting 'within its mandate'. The theory that we have laid out here suggests that Draghi was also lucky, since not all necessary elements for acting as LOLR were present in the euro area 2012 debt crisis, although the germ of all of them was there.

In economies where the borrowing country can commit to satisfy the payments of its newly issued debt until a new issuance is due (as in Eaton and Gersovitz (1981), et al.), Liu et al. (2022) show that as long as a well-designed Fund can effectively absorb the outstanding long-term debt, there is no default and, therefore, no debt dilution through excessive future debt. In this case, the Fund can act as LOLR. However, governments may lack such commitment capacity, which opens the door to further SFCs. Here we have shown how the Fund can act as LOLR for this more general class of economies, provided it can absorb the Gross Financial Needs (GFN) associated with the issuance of new debt (e.g. roll-over debt), as well as guarantee a *minimum intervention* in the sovereign debt market to ensure its sustainability. In particular, we have shown that given a country risk profile there is a minimum *Required Fund Capacity* (RFC) for a Fund contract, associated with a unique 'optimal' maturity structure, which guarantees its role as LOLR.

However, the RFC associated with Fund contracts may be too large for the Fund, in which case the Central Bank (CB) may be able to absorb the additional debt to satisfy the RFC. In this case, there is a *perfect complementarity* between the Fund and the CB, with the former guaranteeing that there is no fundamental or belief-driven sovereign debt risk, if the RFC is satisfied. The Fund contract design allows the CB to intervene guaranteeing the RFC when both act together as a LOLR.[33] Importantly, the RFC is a *prudential-robust institutional* LOLR design, preventing fundamental and self-fulfilling crises, as well as over-lending, making all sovereign debt safe. This means that, in equilibrium, sovereign debt auctions are also safe and private lenders abstain from over-lending.

By calibrating the economy without the Fund to the Italian economy, we show the welfare gains of having a Fund, in this case necessarily complemented by the CB acting as LOLR, and how the welfare gains are significant, but larger when SFCs do not happen even if they are extremely rare events. Alternatively, our work shows that without a well-designed Fund – complemented with the CB to guarantee the RFC – there is no LOLR, contrary to what it is often claimed (and Thomas Cooley would have dismissed).

Data availability

Data will be made available on request.

Appendix A. The Fund contract in recursive form

In this section, we derive the recursive formulation of the Fund contract.

Using the approach of Marcet and Marimon (2019), we say that $c(\iota,x,b_l)$, $n(\iota,x,b_l)$, $\nu_b(\iota,x,b_l)$ and $\nu_l(\iota,x,b_l)$ are a solution to the Fund's contacting problem in *recursive form*, given $b_{l,0}$, if there exists a Fund's value function $FV(\iota,x,b_l)$, transfer policies $\tau_p(\iota,x,b_l)$ and $\tau_f'(\iota',x,b_l)$, with associate private lending policy $b_l' = B_l(\iota,x,b)$ satisfying (2), such that:

$$\begin{aligned} FV(\iota,x,b_l) = \mathcal{SP} \min_{\{\nu_b,\nu_l\}} \max_{\{c,n\}} \; & x\big[(1+\nu_b)U(c,n) - \nu_b V^D(s)\big] \\ & + \big[(1+\nu_l)\tau - \nu_l(\theta^- Z + b_l)\big] + \tfrac{1+\nu_l}{1+r}\mathbb{E}\big[FV(\iota',x',b_l')\big|\iota\big] \\ \text{s.t. } & \tau = \theta f(n) - c \text{ and } x' = \tfrac{1+\nu_b}{1+\nu_l}\eta x \text{ with } x_0 \text{ given,} \end{aligned} \tag{A.1}$$

where $\iota \equiv \{\theta^-,\gamma,\rho\}$ and x' corresponds to the prospective Pareto weight of the sovereign relative to the two lenders with $\eta \equiv \beta(1+r) < 1$, and ν_b, ν_l as the normalized multipliers attached to the sovereign's and the Fund's limited enforcement constraint, respectively.[34] The value function of the contracting problem satisfies

$$\begin{aligned} & FV(\iota,x,b_l) = xV^b(s,x,b_l) + V^l(\iota,x,b_l), \text{ with} \\ & V^b(s,x,b_l) = U(c,n) + \beta\mathbb{E}[V^b(s',x',b_l')|s] \quad \text{and} \quad V^l(\iota,x,b_l) = \tau + \tfrac{1}{1+r}\mathbb{E}[V^l(\iota',x',b_l')|\iota]. \end{aligned}$$

[32] "Within our mandate, the ECB is willing to do whatever it takes to preserve the euro, and believe me it will be enough." Mario Draghi at the London School of Economics.

[33] As we note at the end of Section 7, if the RFC cannot be covered with the joint intervention of the Fund and the CB, a LOLR intervention will require the participation of a Treasury with additional fiscal capacity.

[34] The normalization of the Pareto weights is the same as the one in Ábrahám et al. (2022) and Liu et al. (2022).

We obtain the optimal consumption and leisure policies, $c(\iota, x, b_l)$ and $n(\iota, x, b_l)$ by taking the first-order conditions of problem (A.1),

$$u_c(c) = \frac{1+\nu_l}{1+\nu_b}\frac{1}{x} \quad \text{and} \quad \theta f_n(n) = \frac{h_n(1-n)}{u_c(c)}.$$

This results in a transfer policy $\tau(\iota, x, b_l)$ which corresponds to the lending policy the Fund computes and announces every period.

The prospective relative Pareto weight, x', evolves according to the binding constraints. Particularly, it increases when the sovereign's constraint binds (i.e. $\nu_b > 0$) and decreases when the lender's constraint binds (i.e. $\nu_l > 0$). In the former case, the sovereign's consumption increases not to generate default incentives, while in the latter case, the sovereign's consumption decreases to avoid expected losses from the Fund's perspective.

Appendix B. Proofs

Proof of Proposition 1. The proof follows the argument of Thomas and Worrall (1994) and Zhang (1997). Let's start with fundamental-driven defaults. The participation constraint of the borrower ensures that the value of the borrower is at most equal to its outside option. That is

$$W^b(s^t) = \mathbb{E}\left[\sum_{j=t}^{\infty} \beta^{j-t} U(c(s^j), n(s^j)) \middle| s^t\right] \geq V^D(s^t).$$

For a default to be optimal, it should be that $W^b(s^t) < V^D(s^t)$ which would directly violate the participation constraint. Hence, there is no fundamental-driven defaults.

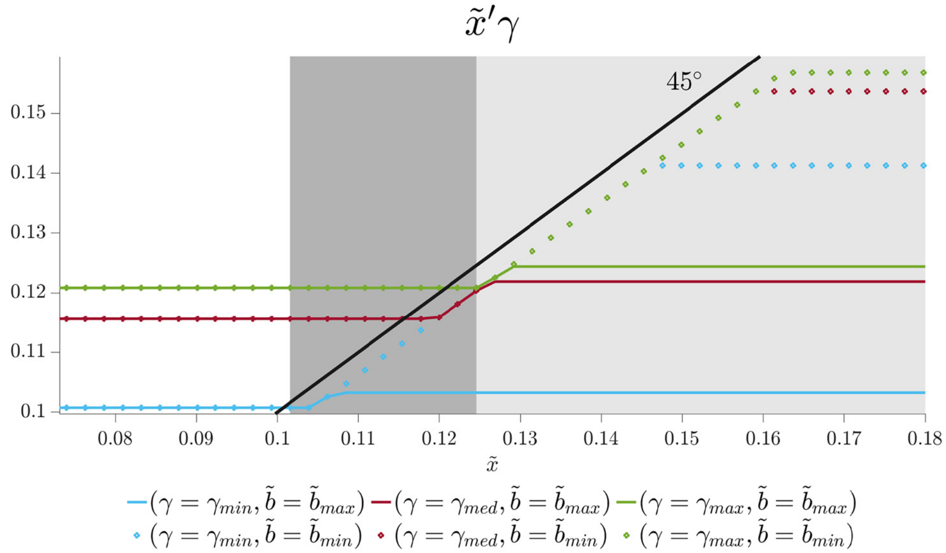
Now consider belief-driven debt defaults. The same argument applies as before with the only difference that the Fund should be potentially capable of providing the entire new debt $\bar{\omega}_{l,t+1}$. Given Definition 2 Part II, the Fund stands ready to absorb the sovereign's entire debt meaning that there is no belief-driven defaults. □

Proof of Corollary 1. Proposition 1 states that in no state there is a default irrespective of the realization of ρ. Hence, by definition of the safe zone in Section 4, the sovereign is always in the safe zone with the Fund's intervention given by Definition 2. □

Appendix C. Additional figures

In this section, we provide an additional figure not presented in the main text.

Fig. C.1 depicts the ergodic set of relative Pareto weights for the Fund without SFC. Compared to Fig. 1, we see that the ergodic set without SFC is related to a higher lower bound for the borrower. Moreover, the upper and lower bound



Note: The figure depicts the law of motion of the detrended relative Pareto weight in the economy without SFC for different growth states and different private debt levels. The y-axis corresponds to the prospective relative Pareto weight, while the x-axis represents the current weight. The dark grey region is the ergodic set which defines the steady state of the economy. The light grey region is the basin of attraction of the ergodic set.

Fig. C.1. Evolution of the relative Pareto weight in steady state in the Fund without SFC.

are closer from each other; as a result, relative Pareto weights fluctuate less, from a higher lower bound, in the economy without SFCs. As noted, these subtle differences in the ergodic sets result in non trivial differences of Table 4.

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