

# Commitment in the Canonical Sovereign Default Model

Xavier Mateos-Planas, Sean McCrary, José-Víctor Ríos-Rull and Adrien Wicht<sup>‡</sup>

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## Abstract

We study the role of lack of commitment in the canonical incomplete-markets sovereign default model of [Eaton and Gersovitz \(1981\)](#). We show the very different set of functional equations that appear under commitment relative to the standard ones. We document how in the standard yearly specification of [Arellano \(2008\)](#) with short-term debt there is no default if there is commitment to the circumstances of when to default. A bad enough disaster makes default under commitment appear. In contrast, with long-term debt, in the standard quarterly [Chatterjee and Eyigungor \(2012\)](#) environment commitment only to one-period-ahead default barely changes the no-commitment allocation, but commitment to both the one-period-ahead default circumstances and the one-period-ahead dilution, or commitment to a longer horizon (a year or a bit more) eliminates default completely and it is equivalent to commitment in the one-period-ahead default with short-term debt.

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\*Mateos-Planas: Queen Mary University of London and CfM; McCrary: The Ohio State University; Ríos-Rull: University of Pennsylvania, University College, London, NBER, CEPR, and CAERP; Wicht: University of Basel.

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# 1. Introduction

One of the canonical models in economics is the [Eaton and Gersovitz \(1981\)](#) model of sovereign default. This model incorporates two key frictions: first, the set of financial instruments is coarse (debt is the only security and must either be paid in full or defaulted on entirely), and second, the circumstances under which default occurs are not pre-specified in the contract but are instead determined unilaterally by the borrower after the fact.

This paper assesses the importance of the second friction—that the borrower lacks commitment to the circumstances of default—and its role in shaping the outcomes in the sovereign default environment for both short-term and long-term debt. Addressing this question is relevant in practice. A priori, as plausible as the usual assumption of no-commitment is the idea that in debt markets there is an acknowledged understanding of the repayment rules the sovereign is committed to (implicitly enforced by arbitrarily severe threats or reputation). While existing work has looked at it to some extent, we intend to explore the possibilities of this idea within the main workhorse of the sovereign debt literature.<sup>1</sup> At a normative level, this is also potentially very important if we think that international institutions and creditors (the IMF, the Paris Club, the London Club, and others) can help reshape the international environment both creating sophisticated state contingent securities (and hence helping get around the current coarseness of those available) or affecting the enforceability of default clauses agreed in such contracts by changing the punishments to deviating from those default rules.<sup>2</sup> We want to contribute to identify the areas of highest return of these activities.

To examine this, we develop versions of the model with commitment to specific default conditions (under which realizations of economic shocks default can occur) and compare their properties to those of the Markov equilibria in the standard no-commitment model and also to models with other specifications of commitment.

We demonstrate that the equilibrium under commitment is characterized by a substantially different pair of functional equations compared to the standard model. Notably, the indifference threshold between default and non-default is no longer relevant, and the sovereign commits to repay even in situations where it would prefer to default ex-post. This commitment is made to secure better loan terms.

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<sup>1</sup> Existing work includes, for instance, [Adam and Grill \(2017\)](#), [Roettger \(2019\)](#), and, following the seminal [Grossman and Van Huyck \(1988\)](#) on excusable defaults, [Alfaro and Kanczuk \(2005\)](#) or [Daniel and Nam \(2022\)](#).

<sup>2</sup> There is a revived interest in state-contingent debt instruments by the IMF and World Bank, e.g., [IMF \(2017\)](#), [Roch and Roldán \(2023\)](#). Evidence in [Schumacher et al. \(2021\)](#) documents the increasing proliferation of legal litigation on the part of creditors, thus making the breach of pre-agreed conditions more costly. See also [Gelpert and Panizza \(2022\)](#).

With short-term debt, commitment means that default conditions are determined one period in advance, and therefore fully pin down the current equilibrium price of debt which allows treating the equilibrium problem as a maximization (or Ramsey) problem. For long-term debt, we consider two primary interpretations of commitment: first, as commitment only to the circumstances of default, and second, as commitment to both the circumstances of default and to a debt repurchase price rather than a specific quantity of debt issued. The first interpretation still exhibits time inconsistency and requires solving for an equilibrium with its associated fixed point problem. The second interpretation, however, reduces to a straightforward maximization problem. This second type of commitment implies the same allocation as that that results with short-term debt and commitment to default.

To explore the quantitative relevance of commitment versus non-commitment we analyze the two canonical environments in the literature for short-term and for long-term debt: the baselines of [Arellano \(2008\)](#) and of [Chatterjee and Eyigungor \(2012\)](#). The well known problem of existence and computability of the latter induced us to pose the problem with extreme value shocks to the default choice which is the modern way of bringing randomization to smooth choices.

We find that commitment in the [Arellano \(2008\)](#) environment of short-term debt eliminates default. Moreover, it leads the very impatient sovereign (implied by the paper's calibration) to borrow massively larger amounts, with debt occasionally at values just below the conventional natural debt limit but never exceeding it. Commitment does not logically eliminate the possibility of default but it effectively does so, a result that appears to be robust to parametric changes. In these economies, the cost today of rolling over debt from committing to some default tomorrow overcomes its consumption smoothing benefits tomorrow.<sup>3</sup> We also find that there is an occasional logical possibility of the sovereign attaining levels of debt that are larger than the natural debt limit (that one that is feasible to be paid off with probability one). We do so by finding that the range of the debt holdings reach that far. However, while this scenario is logically possible, it demands such an improbable series of bad shocks that it did not materialize even after simulating the economy for 15,000,000 periods. We can achieve default (in ways similar to the spirit of [Adam and Grill \(2017\)](#)) by postulating disaster type states. This can happen explicitly with sizable probability if we carefully construct probability distributions that include a moderate yet non-negligible probability of a bad enough state.

In the [Chatterjee and Eyigungor \(2012\)](#) environment with long-term debt we find that commitment only to default circumstances has small effects, providing the sovereign with somewhat better terms

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<sup>3</sup> Conceivably, an expansion of the support of the income process could work towards more default. It is thus that we have considered a discretization of the standard AR(1) process à la Tauchen that includes a sufficiently bad state (5 standard deviations below the mean) even if it has an extremely low probability. In this case the sovereign attains levels of debt that exceed the natural debt limit (the one that is feasible to be paid off with probability one), yet the occurrence of default remains elusive.

that it uses to borrow more which in turn leads to more default. Commitment to both the terms of default and to the continuation price (which is a form of weighing the quantity of debt issued and the outlook of the economy) eliminates default from the environment. However, if the commitment just to not default is extended to a year (by calibrating the model to yearly periods) or the debt is shortened to last a year, commitment almost eliminates default (not quite it goes from 9% per year to less than 1%).

We conclude from this observation that commitment is very important but it cannot be limited to the terms of default for a short period, it has to include commitment to avoid debt dilution or extend for longer periods. This is clearly the case in the the quarterly model of [Chatterjee and Eyigungor \(2012\)](#). In other words we not only highlight the importance of pre-specifying the terms of default and being able to commit to them, but also that is not enough. Commitment to not dilute the debt is also necessary. An important issue is the length of the period as the commitment to not default is only for one period. While the [Chatterjee and Eyigungor \(2012\)](#) economy is set to have quarters as periods, a recalibration of the model so that periods are years or the use of shorter debt (no much longer to a year) makes commitment only to default to be almost as useful as commitment to default and to not dilute.

There are some additional contributions of the paper. First, we show how to pose the model with long-term debt and extreme value shocks with and without commitment explicitly, which has the mild difficulty of computing the utility under commitment of taking advantage of the extreme value shocks to choose when to default. We also point out a feature of extreme value shocks that provide additional punishment when defaulting if choosing a location parameter of zero (this is due to the fact that the expected value of the utility of the shocks when choosing to default or not is  $\alpha(\gamma + \ln 2)$  and that is given up upon defaulting) with the implicit additional punishment when defaulting introduced if not properly choosing the value of the location parameter. Second, we assess the monotonicity of the default decisions when the output shocks are persistent and when punishment is temporary. We can do so because we pose extreme value shocks to the default decision which transforms the problem from looking for a default threshold to looking for default probabilities for each realization of output. When there is not default (up to computer precision) this is a feature for all possible values of output and when there is default the probability of this happening is both steep and monotonic.

We are not the first paper that analyzes the role of commitment in the sovereign default environment with short-term debt. [Adam and Grill \(2017\)](#) pose a model where a government with commitment solves for the optimal allocation (they call it a Ramsey plan) using short-term debt. Their environment is different to the standard one: partial default is possible at an output cost that is

proportional to the fraction defaulted (vs. a zero-one decision that translates into a temporary exclusion from borrowing). Indeed, in their environment, perfect consumption smoothing is possible. They also find that default does not happen. They pose a disaster like shock (low probability yet persistent devastating outcome) to obtain default. Their economy is very stylized with only two realizations of the shock and it is not immediately relatable to the standard papers in the literature.

Among the recent still unpublished work, perhaps the paper closest to ours is [Roettger \(2019\)](#) who studies the role of partial commitment à la [Debortoli and Nunes \(2010\)](#) in the standard [Arellano \(2008\)](#) environment such that having commitment or discretion in a given period is random. His analysis limits the maximum debt held to the endogenous upper bound of debt holdings that appear in [Arellano \(2008\)](#) (a bound that is not logically justified and that our work indicates that is surpassed by an enormous margin) and find that default does not happen.<sup>4</sup>

[Aguiar et al. \(2019\)](#) study the determination of maturity by an indebted sovereign without commitment in a model where the only uncertainty is the value of default. This means that the shocks have no lingering effects beyond the default choice, they do not affect either consumption nor subsequent debt holdings unlike in the standard model. They note that if the borrower could commit to the states in which default occurs then allocations would not be affected by the maturity of bonds. Under the same type of commitment, we instead find that outcomes with short-term debt and long-term debt differ, unless there is also commitment to the debt repurchase price.

[Hatchondo et al. \(2020\)](#) study commitment to the path of sovereign debt issuance, rather than commitment to default as we do, and find that issuing long-term debt is different from issuing short-term debt, and that with short-term debt commitment and no-commitment are equivalent. With commitment to default, we instead have shown that short-term and long-term may be equivalent but only if there is also commitment to the repurchase price of debt, and that commitment and no-commitment have sharply contrasting implications, even with short-term debt. They consider, as we do, long-term debt, but their notion of commitment is different than the two interpretations that we explore (and from those that imply some form of state contingent securities) so we find our work complementary to theirs.

Finally, [Pouzo and Presno \(2022\)](#) study optimal taxation in a risk-neutral closed economy with only short-term government debt, and without commitment to debt repayments. Although their question and model are thus different from ours, for comparison they also consider a commitment-to-default version and find that having one-period-ahead commitment to default is isomorphic to having

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<sup>4</sup> He obtains default when he changes risk aversion to 10 while keeping the rest of the environment unchanged. We replicate this finding with the upper debt limit that he poses. There is no default, however, with our upper debt limit.

commitment to the full path of default. We have also found a similar equivalence but in a very different context, and additionally study long-term debt. Also analyzing optimal taxation, [Clymo and Lanteri \(2019\)](#) model partial commitment as the ability to commit only for a finite horizon. They propose a sufficient condition for the equivalence between outcomes in full and partial commitment. The idea is that when certain future variables that affect the current constraint set are predetermined for a sufficiently long period, time-inconsistency issues can be resolved. We complement their work by showing that with long-term debt commitment to a longer horizon eliminates default completely.

To save space we pose the model in its general form (with the possibility of long-term debt and with extreme value shocks) with and without commitment and compare their properties, in [Section 3](#). The standard short-term debt without extreme value shocks is a special case. [Section 4](#) explores quantitatively the prevalence of default, first for short-term in the canonical specification of [Arellano \(2008\)](#) where default does not happen ([Section 4.1](#)), then in examples where default starts appearing ([Section 4.2](#)), finally in the canonical long-term debt economy of [Chatterjee and Eyigungor \(2012\)](#) ([Section 5](#)).

## 2. The Fundamental Difference between Commitment and Not-Commitment

Under no-commitment, the sovereign chooses how much to save or borrow and whether to default, unrestricted by the past. Under commitment, the sovereign honors its past obligations about defaulting now and chooses how much to save or borrow and the circumstances under which it will default. In the case of long-term debt commitment requires a more precise definition as we will see below. Consequently, the optimality conditions differ depending on whether commitment exists. Lack of commitment implies that the choice made gives the highest utility (and because of continuity) this yields that the choice threshold satisfies an indifference condition. This is not the case under commitment, where the optimality condition can be heuristically described by stating that the indifference condition between defaulting and not defaulting takes also into account the increased loss in the previous period consumption that comes from a more pervasive default.

When there is only short-term debt the lack of commitment to future borrowing amounts or defaulting circumstances beyond the following period has no relevance, but it matters a lot for long-term debt where there is still some form of time inconsistency because of dilution. Consequently, with long-term debt there are various possible interpretations to the meaning of commitment:

1. **Commitment Only to Default Probabilities** The sovereign can commit to the circumstances of default the following period, but not to its continuation policies after that in any form, either

to future borrowing amounts or future defaulting circumstances.

2. **Commitment to Default Probabilities and to an Amount of Dilution (to the Debt Repurchase Price)** The sovereign can commit to the circumstances of default and to a continuation price of the debt that is not state contingent. This is the way to commit to the extent of dilution. This option implies the same allocations with short and long-term debt as is as if the sovereign issued long term debt and commits to repurchase it at a pre-specified price (a point made already in the introduction of [Aguilar et al. \(2019\)](#)) that we prove in the Appendix. This problem can be written as a Ramsey type problem with the price of debt (the default probability) restricted to being measurable with respect to the history up to the previous period.
3. **Commitment to Default Probabilities and Expected Value of the Debt Repurchase Price** This means that the sovereign can issue state contingent securities as its price the following period is allowed to vary as long as it respects the expected value. This problem can be written as a Ramsey type problem with the price of debt or default probability being allowed to being measurable up to the period where it is being traded and is akin to a problem with full state contingent securities. We do not analyze it quantitatively because it can be trivially used to achieve constant consumption.
4. **Commitment to Debt Issuances if Not Defaulting** The sovereign can commit to the continuation debt path contingent on not defaulting as a function of the endowment, but not to whether it does default or not (the object of [Hatchondo et al. \(2020\)](#)). This is complementary to our work.

Of these possible interpretations of commitment, we are interested in the first two. The last two, while very interesting, imply a form of state contingent debt which lies outside the question that we are after.

### 3. The Model and its Recursive Representations.

We pose the model with common elements to the various versions of the model that we study (short-term debt and long-term debt) with the addition of smoothing shocks to eliminate the well known ([Chatterjee and Eyigungor \(2012\)](#), [Mateos-Planas et al. \(2024\)](#)) problems of convergence associated to long-term debt. We choose extreme value shocks to the value of defaulting and non defaulting rather than the original truncated normal shocks to the endowment given their good properties and their recent popularity in the literature ([Mihalache \(2024\)](#), [Dvorkin et al. \(2021\)](#)). Moreover, to avoid cumbersome notation, we pose the shocks as iid and the punishment to be perpetual. Nothing theoretically relevant

is lost with these assumptions, and in our quantitative work, endowment shocks are Markovian and the punishment from defaulting involve temporary stochastic exclusion from borrowing as is standard.

An infinitely lived agent, the sovereign, has preferences over discrete streams of consumption, assessed according to a standard utility function  $u(c)$ ,  $c \geq 0$ , and discount rate  $\beta$ . It maximizes expected utility. The sovereign has endowment  $y \in [\underline{y}, \bar{y}]$  with cdf  $F^y$ . Let  $\bar{F}^y(y) = 1 - F^y(y)$ .

The sovereign has access to a market of unsecured debt where it can borrow. Competitive lenders are risk neutral and demand and obtain an expected return  $r^*$  implying an inverse expected return  $q^* = \frac{1}{1+r^*}$ . Every unit borrowed involves a promise to pay one unit the following period, and a coupon  $\lambda^{t-1}$ ,  $\lambda < 1$ , all subsequent periods. This implies that one unit of debt issued in period  $t$  is identical to  $\lambda^{-t}$  units issued  $t$  periods before. Let  $\bar{\lambda} = 1 - \lambda$ . We denote by  $b$  the coupon due and by  $q$  the price of a newly issued coupon. The promise to pay is either honored or not, and, if not, the sovereign cannot save or borrow ever again yielding a value of defaulting of  $v^a(y) = u(y) + \frac{\beta}{1-\beta} \int u(y') dF^y(y')$ . Every period, prior to the defaulting action the sovereign draws two iid Gumbel  $(\mu, \alpha)$  random variables  $\{\epsilon^a, \epsilon^r\}$  that add to its utility depending on the action taken (choosing autarky or repayment). Given our assumptions,  $\lambda = 0$  implies short-term debt, and  $\alpha = 0$  the absence of extreme value shocks which is the case in [Arellano \(2008\)](#).

### 3.1. Lack of Commitment

Without commitment the sovereign is free to default as it wishes and to issue any amount of debt that it sees fit. Moreover, the problem of the sovereign is not a maximization but an equilibrium relation with its future selves. The literature focuses its analysis on the Markov equilibria in this environment and analyzes the problem with recursive methods. The state of the economy is the pair endowment  $y$ , and coupon due  $b$ . We denote the relevant objects with superindex  $n$ .

**Short-Term Debt without Extreme Value Shocks** Because this problem has not displayed computational difficulties, it has been posed without extreme value shocks ( $\alpha = 0$ ) which simplifies somewhat its characterization. The state is the pair  $\{y, b\}$  and to characterize the problem as a Markov equilibrium it suffices to note that the sovereign takes as given the default location  $d^n(b')$  (in this environment the default set is a threshold [[Arellano \(2008\)](#), [Clausen and Strub \(2020\)](#)]) and savings choice  $g^n(y', b')$  policies followed by the sovereign's successor and chooses whether to default, and, if not, how much to consume and save/borrow. Lenders also take as given the sovereign's successor policies which in turn implies the equilibrium condition for the price of debt:  $q^n(b') = q^* \bar{F}^y[d^n(b')]$ .



These considerations imply that the sovereign's problem under repayment is

$$v^n(y, b) = \max_{b'} \left\{ u \left[ y - b + q^* \bar{F}^y [d^n(b')] \right] + \beta \left[ \int_{\underline{y}}^{d^n(b')} v^a(y') dF^y(y') + \int_{d^n(b')}^{\bar{y}} v^n(y', b') dF^y(y') \right] \right\}. \quad (1)$$

The solution to this problem (after imposing the equilibrium condition that the sovereign's choices end up being equal to the decision of its follower) is a pair of functions for saving,  $g^n(y', b')$  and default location,  $d^n(b')$ , that solve (1) and require that the default thresholds make the sovereign indifferent between defaulting and paying back:

$$v^a(d^n(b)) = v^n(d^n(b), b). \quad (2)$$

**Long-Term Debt with Extreme Value Shocks** The equilibrium of the long-term debt without commitment has (not only) computational difficulties and researchers have responded with the use of randomization. We have chosen Gumbel shocks to both default and not default, which makes the defaulting choice probabilistic. Consequently, the sovereign takes as given the probability with which the future sovereign will repay at each realization of the endowment shock  $\zeta^n(y', b')$ , and the savings choice  $g^n(y', b')$ . Lenders also take those functions as given which implies that the price of debt is  $q^n(b') = q^* \int [1 + \bar{\lambda} q^n[g(y', b')]] \zeta^n(y', b') dF^y(y')$ . In a Markov equilibrium the sovereign choices yield decision rules that coincide with those that it expects future sovereigns to follow. Taking advantage of the properties of extreme value Gumbel shocks, these considerations imply the value of repayment is

$$\begin{aligned} v^n(b, y) &= \max_{b'} \left\{ u \left[ y - b + q^n(b') (b' - \bar{\lambda} b) \right] + \beta \int \left[ \int \max \{ v^a(y') + \epsilon^a, v^n(b', y') + \epsilon^r \} dF^\epsilon(\epsilon^a, \epsilon^r) \right] dF^y(y') \right\} \\ &= \max_{b'} \left\{ u \left[ y - b + q^n(b') (b' - \bar{\lambda} b) \right] + \beta \int \alpha \ln \left( e^{v^a(y')/\alpha} + e^{v^n(b', y')/\alpha} \right) dF^y(y'), \right\} \end{aligned} \quad (3)$$

where the repayment probability is

$$\zeta^n(y', b') = \frac{e^{v^n(y', b')/\alpha}}{e^{v^a(y')/\alpha} + e^{v^n(y', b')/\alpha}}. \quad (4)$$

### 3.2. Commitment

With short-term debt the meaning of commitment is clear: the specification of under what circumstances to default tomorrow is made today. As stated before with long-term debt we have two notions of commitment.

**Short-Term Debt without Extreme Value Shocks** Following the sovereign debt literature, we pose the problem of the sovereign using recursive language, but we should note (we provide a proof in the Appendix) that this is a maximization problem, indeed a Ramsey problem that can also be solved using non-recursive, sequential methods. Also, following the literature we pose this problem without extreme value shocks as a threshold problem, but because we also solve it with extreme value shocks, we always verify that the solution is a threshold in the economies that we solve. The problem of the household is:

$$v^c(d, b, y) = \begin{cases} v^a(y), & \text{if } y < d. \\ \max_{d', b'} \left\{ u \left[ y - b + q^* \bar{F}^y(d') \mid b' \right] + \beta \int v^c(d', b', y') dF^y(y') \right\}, & \text{if } y \geq d, \end{cases} \quad (5)$$

where  $c^c(d, b, y)$  is the consumption policy. The solution satisfies the first order conditions (if  $y \geq b$ ) (in addition to Equation (5) and the budget constraint):

$$\bar{F}^y[d^c(d, b, y)] q^* u_c[c^c(d, b, y)] = \beta \int_{d^c(y, b)} u_c[c^c(d', b', y')] dF^y(y'), \quad (6)$$

$$g^c(d, b, y) q^* u_c[c^c(d, b, y)] = \beta (v^a(d^c(d, b, y)) - v^c(d^c(d, b, y), g^c(d, b, y), d^c(d, b, y))). \quad (7)$$

Strict concavity implies that the solution is unique. Note that Equation (7) is very different than the indifference condition (Equation (2)) at the threshold that appears in the environment without commitment where the condition is that the right hand side is zero.

**Long-Term Debt and Extreme Value Shocks** We start with the case that the sovereign can commit only to the probability of default one period ahead. With extreme value shocks and commitment to a probability of default, the sovereign chooses to implement that probability in the most advantageous circumstances. In our case, this requires that we are explicit of how the choice of probabilities

of default is transformed into utility. Let  $v^{c*}(b, y)$  and  $v^a(y)$  be the values of repaying and defaulting that exclude current period's associated realization of the extreme value shocks. Function  $v^{c*}$  is the result of repeatedly applying the equilibrium decision rules for repayment probabilities  $\zeta^{c*}(b, y)$  and for savings  $g^c(b, y)$ . Then the properties of extreme value shocks imply that a sovereign today that does not default and chooses how much to issue today  $b'$  and the probabilities of repaying tomorrow  $\zeta(y')$  gets

$$\begin{aligned} W(b, y, \zeta(\cdot), b') = & u[y - b + q[\zeta(y'), b'] (b' - \bar{\lambda}b)] \\ & + \beta \int \left\{ \zeta(y') v^{c*}(b', y') + [1 - \zeta(y')] v^a(y') + \right. \\ & \left. \alpha \left[ \ln \left( \frac{1}{1 - \zeta(y')} \right) + \zeta(y') \ln \left( \frac{1 - \zeta(y')}{\zeta(y')} \right) \right] \right\} dF^y(y'). \end{aligned} \quad (8)$$

Consequently, the problem of the sovereign is to maximize  $W(b, y, \zeta(\cdot), b')$  with respect to  $\zeta(y')$  and  $b'$ . The equilibrium requires that the chosen policies equal to  $\zeta^{c*}(b, y)$  and  $g^c(b, y)$  and of course that  $v^{c*}(b, y) = W(b, y, \zeta^{c*}(b, y), g^*(b, y))$ .

Commitment with long-term debt to the repayment probabilities and to the dilution amount turns out to be equal to commitment to the default probabilities with short-term debt, a result that we prove in the Appendix. This is not a surprising result, as committing to the terms of dilution implies commitment to a debt price which in turn can be achieved by buying all the debt at a pre-specified price which in turn is just short-term debt.

## 4. Quantitative Analysis with Short-Term Debt

We examine how resource allocation and default rates differ under commitment when using short-term debt. Our analysis proceeds in three steps. First, we use the standard specification of short-term debt from the literature, the [Arellano \(2008\)](#) economy where commitment prevents default ([Section 4.1](#)). Second, we consider alternative specifications with short-term debt where default occurs ([Section 4.2](#)). Finally, we analyze commitment in the canonical model of long-term debt from [Chatterjee and Eyigungor \(2012\)](#) ([Section 5](#)).

### 4.1. The [Arellano \(2008\)](#) Economy

We extend the model presented in [Section 3](#) with key elements from [Arellano \(2008\)](#) (an economy calibrated for short-term debt with annual periods): a persistent Markovian endowment with cumulative

distribution  $F^y(y'|y)$  and density  $f(y'|y)$ , a direct cost to defaulting such that consumption in autarky is  $c = h(y) \leq y$ , and a probability  $\theta$  of exiting autarky. The log of the endowment follows a zero-mean AR(1) process with normally distributed innovations. We maintain all functions, parameter values and endowment grid specifications from [Arellano \(2008\)](#).<sup>5</sup>

A crucial consideration is the upper bound on debt. While [Arellano \(2008\)](#) set a non-binding upper bound of 1.5 times mean endowment,<sup>6</sup> we extend this bound in the commitment economy to exceed both the natural debt limit (the maximum debt allowing positive consumption and repayment in all states, equal to  $(1+r)\underline{y}/r > 0$ , or about 50 times mean endowment) and any attainable debt levels.

Following [Arellano \(2008\)](#), we solve the no-commitment version through state space and choice set discretization. We use the exact same grid for assets as in that paper, with its even size of 0.25% of average income. [Table 1](#)'s first column shows our implementation of the [Arellano \(2008\)](#) economy. The main parameters and debt statistics (in average GDP units) reveal very low average debt relative to the natural borrowing limit, with no positive asset positions. The economy defaults 3% of the time, with 5% spreads, consistently with the original paper's results.

We turn now to the economy with commitment. Using the above grid specification, default is never chosen, and the lower asset bound (credit limit) becomes binding. We extend this lower bound up to the natural limit so it becomes non-binding and, to detect any possibility of default in the vicinity of the natural limit, we specify a much finer grid for debt of size 0.01% of average output (25 time finer than in the [Arellano \(2008\)](#) original setting). The second column of [Table 1](#) presents our key findings for the allocation properties under commitment. Two crucial results emerge: default never occurs, and debt levels become enormous, with approximately 80% of output servicing interest payments. While debt approaches the natural borrowing limit, it never exceeds it.<sup>7</sup>

Alternative parameterizations yield similar patterns. Default remains absent across specifications. Higher discount rates or risk aversion (Economies (i) through (iii)) modestly reduce average debt and increase ergodic distribution consumption. Even with risk aversion of 10 (as in [Roettger \(2019\)](#)), we

<sup>5</sup> The endowment process follows  $\log y' = \rho_y \log y + \epsilon'$ ,  $\epsilon' \sim N(0, \sigma_\epsilon)$ , with unconditional mean endowment of 1. We approximate it via [Tauchen \(1986\)](#) discretization method with  $n_y = 21$  points, setting the lowest output value at minus 3 standard deviations from a normalized mean of zero. We specify  $h(y) = \min\{1 - \pi, y\}$ , with parameters  $\beta = 0.95285$ , risk aversion of 2, risk free rate of .017,  $\rho_y = 0.945$ ,  $\sigma_\epsilon = 0.025$ ,  $\theta = 0.282$  and  $\pi = 0.031$ . The implied natural limit for debt is  $b^n \equiv (1+r)\underline{y}/r = 47.419$ .

<sup>6</sup> [Roettger \(2019\)](#) uses a bound that is also suitable for the economy without commitment (it is even tighter), which proves significant. Their extensive explorations yield default under commitment with risk aversion of 10. The default appears to be due to the debt limit.

<sup>7</sup> In one of the economies that we studied, the one with risk aversion of 10, the decision rules show the possibility of crossing this default threshold. However, the sequence of extremely bad shocks that are needed for this to happen never occurred in our simulation of 15,000,000 periods.

observe lower debt and higher consumption. We also consider parameter combination recalibrating the original [Arellano \(2008\)](#) model with increased risk aversion, showing debt reductions and consumption increases. The table also documents effects from increased innovation variance and various output autocorrelations without much change in the outcomes.

**Table 1: Commitment in the [Arellano \(2008\)](#) Economy**

	NoComm	Comm	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
<b>Parameters</b>									
$\beta$	0.953	0.953	0.975			0.918			
Risk Aversion	2.000	2.000		5.000	10.0	5.000			
$\sigma_\epsilon$	0.025	0.025					0.045		
$\rho_y$	0.945	0.945						0.925	0.965
<b>Distrib of assets <math>b</math></b>									
aver $b$	<b>0.05</b>	<b>47.02</b>	44.99	43.20	37.37	45.43	38.21	48.58	44.36
min $b$	0.00	44.68	34.17	35.57	23.04	41.95	32.66	46.17	43.13
max $b$	0.25	47.42	47.42	46.78	44.68	47.40	39.20	49.00	44.72
nat borrow lim $b$	47.42	47.42	47.42	47.42	47.42	47.42	39.20	49.00	44.72
<b>Time-series stats</b>									
<i>Means</i>									
freqdef	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
debt to $y$	0.04	47.32	45.32	43.51	37.67	45.74	39.04	48.80	44.80
tb/ $y$	0.00	0.79	0.75	0.72	0.63	0.76	0.65	0.82	0.75
spread	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>Standard deviations</i>									
spread	0.075	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
log $y$	0.078	0.08	0.08	0.08	0.08	0.08	0.14	0.07	0.10
log $c$	0.080	0.44	0.34	0.28	0.21	0.32	0.44	0.41	0.47
tb/ $y$	0.013	0.06	0.06	0.06	0.06	0.05	0.09	0.05	0.07
<i>Correl. with log output</i>									
spread	-0.35	—	—	—	—	—	—	—	—
log $c$	0.98	0.94	0.74	0.77	0.60	0.90	0.92	0.91	0.95
tb/ $y$	-0.11	-0.97	-0.59	-0.60	-0.26	-0.86	-0.89	-0.93	-0.99

The first column 'NoComm' is the original [Arellano \(2008\)](#) Economy. The second column 'Comm' corresponds is the same Environment with Commitment. The remaining columns (i) to (vii) consider various changes of parameters in the commitment model. Top panel: parameters. Medium panel: Moments for assets in the simulated ergodic distribution. Bottom panel: Standard statistics from the simulated time-series. Models are simulated for 15 Million periods, dropping the initial 10,000 realizations for the calculation of moments.

Removing all default penalties except the one-period savings restriction yields consistent results: the sovereign never defaults and maintains debt below the natural borrowing limit.<sup>8</sup>

This implies that under commitment, the equilibrium allocation matches what would occur in a standard incomplete markets economy without default options, where the sovereign must always repay its debt.

<sup>8</sup> Our characterization rules out, by construction, default on the upper range of income realizations. While persistent shocks might theoretically enable default at high income levels (as persistence could reduce the value of market access), our examples show no such cases: one-shot deviations to default at any threshold never dominate choices made with a single lower threshold.

## 4.2. Economies that can Generate Default under Commitment

It is always possible to construct example economies where there is default. It suffices to pose a two period economy with two states in the second period and choose the endowments in both periods so perfect insurance can be implemented via a securities portfolio that perfectly replicates default. This said, we are interested in economies of the type that are used for the sovereign default problem, and for them default under commitment requires some combination of a bad enough state and a low enough probability of the bad enough state.<sup>9</sup>

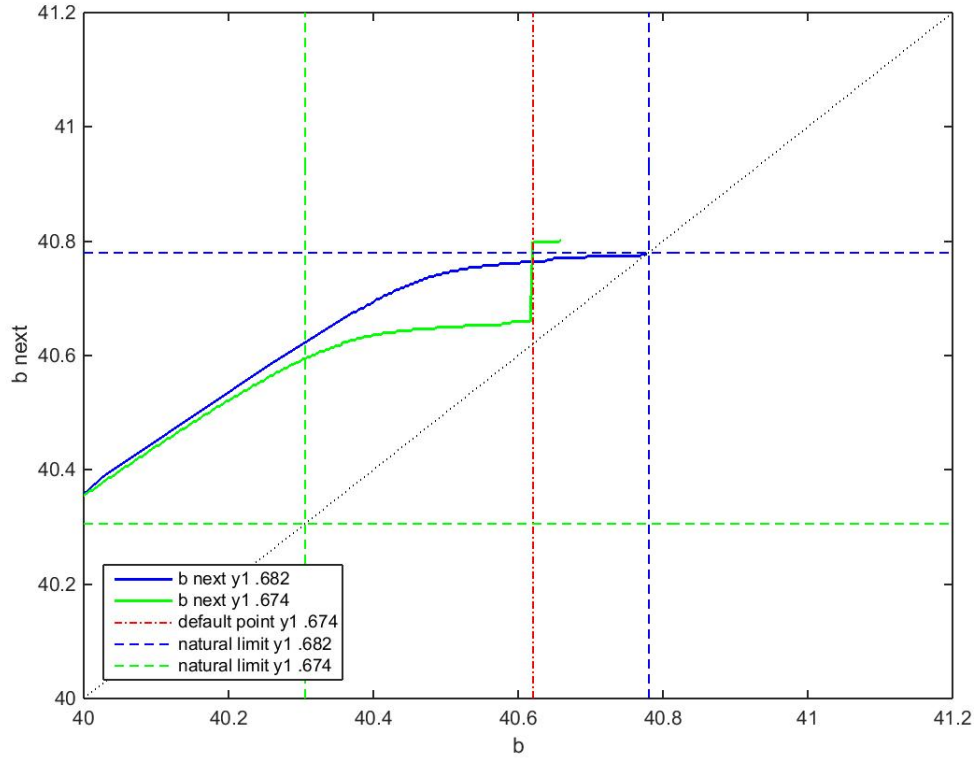
We now proceed to construct an example, where we have a large disaster with a relatively low probability. The economy, like Arellano's, has CRRA preferences with a risk aversion of 2, discount rate  $\beta = 0.95285$ , a risk free interest rate of 1.7%, and no direct costs of default other than current autarky. We pose a grid for debt of size 0.00125 of average endowment. Shocks are i.i.d. with 21 possible states  $y_j$  symmetrically posed around 1 separated by  $x = 0.02073504$  from each other so that  $y_1 = 1 - 10x = 0.7926$  and  $y_{21} = 1 + 10x = 1.207$ . The probability of each one of those points is

$$\gamma_j = \lambda(s) \times \begin{cases} e^{s(y_j-1)} & j \leq 10, \\ e^{-s(y_j-1)} & \text{otherwise} \end{cases}$$

for some  $s \geq 0$  where  $\lambda(s)$  is simply a scaling parameter that ensures that  $\gamma$  is indeed a probability distribution and that coincides with the probability of the mean value of the endowment. Note that the case  $s = 0$  is the uniform distribution and the larger  $s$  is, the more unlikely the extreme states are. Note also that the mean of the process is always one. By scaling  $s$  we look for when default starts occurring. Now let  $s = 20$  which implies a probability of the worst state of 0.33%. Now we reduce the value of  $y_1$  below 0.7926 (while increasing that of  $y_{21}$  to maintain a mean of 1). With a value of  $y_1 = 0.6817$  there is still no default but with a value of 0.6737 default appears. Default only shows up when the level of debt is larger than the natural borrowing limit and is extremely rare, it takes simulations of millions of periods to obtain an occurrence. Obviously we could engineer default with a worst state arbitrarily close to zero but we are interested in giving a sense of when it starts happening.

Figure 1 shows the debt policies for the worst income state in the two economies discussed. The blue line shows that of the economy where there is no default and we see how there is a fixed point (the

<sup>9</sup> In the Arellano (2008) economy, we could construct a Tauchen type approximation with a lower bound low enough that there is default there. However, the probability of such state would be so low as to not happen in any sample of reasonable size (with risk aversion of 10 and the lower bound 5 standard deviations below the mean the decision rule displays default but we obtained no realizations of default in a sample of size 15,000,000). Earlier versions of this paper available on request included another example where we reduced dramatically the probability of the worst state until it is conceivable that default occurs even if extremely unlikely.



**Figure 1: Debt Choices of Two Economies that Differ in the Size of the Disaster State**

sovereign issues the same debt that it had before in the same state) just below its natural borrowing limit. The green one is the bond policy in the economy with default which has a worse state and therefore a tighter natural borrowing limit (discontinuous green line). Default is used to allow the sovereign to extend the debt that it can hold.

## 5. Quantitative Analysis with Long-Term Debt

We take the [Chatterjee and Eyigungor \(2012\)](#) environment as the canonical model with long-term debt. To avoid the well known computational/non-existence problems with long-term debt, [Chatterjee and Eyigungor \(2012\)](#) posed iid income shocks. With the same aim, we follow a more modern strategy and use instead extreme value shocks. In their original paper, [Chatterjee and Eyigungor \(2012\)](#) calibrated the economy using as targets the mean and standard deviation of spreads as well as the debt to output ratio using the discount rate and the linear and quadratic terms of the default cost. We follow the same strategy, adding the correlation of spreads and output and the relative standard deviation of consumption and output as additional targets while adding the variance of the extreme value shocks

Statistics	Targets		Parameter	Description	Value	Values in Chatterjee and Eyigungor (2012)
	Model	Data				
Average spread	0.0816	0.0815	$\beta$	discount factor	0.9564	0.9540
St. Dev of spr.	0.047	0.044	$d_0$	default cost parameter	-0.1407	-0.1882
Debt-to-Y ratio	0.70	0.70	$d_1$	default cost parameter	0.2248	0.2456
corr(spread,Y)	-0.75	-0.79	$\alpha$	taste shock parameter	0.0033	*
$\sigma(c)/\sigma(y)$	1.09	1.09	$\phi$	utility cost of default	0.0035	*

**Table 2: Calibration of the Chatterjee and Eyigungor (2012) Economy**

Variable	Data	Value	Values in Chatterjee and Eyigungor (2012)
$\sigma(c)/\sigma(y)$	1.09	1.09	1.11
$\sigma(NX/y)/\sigma(y)$	0.17	0.14	0.20
$corr(c, y)$	0.98	0.99	0.99
$corr(NX/y, y)$	-0.88	-0.61	-0.44
$corr(r - r_f, y)$	-0.79	-0.75	-0.65

**Table 3: Non Targeted Moments of the Chatterjee and Eyigungor (2012) Economy**

and the cost of default as additional parameters. We use value function iteration, interpolating the values function using Chebyshev polynomials (details in the Appendix).

Table 2 displays the targets and parameter values of our implementation of the Chatterjee and Eyigungor (2012) economy as well as the original ones. We deem our calibration successful. Table 3 displays some non-targeted statistics and their values in the Chatterjee and Eyigungor (2012) economy. This table also shows that our choice of the variance of the shocks improves on the performance of Chatterjee and Eyigungor (2012) that obtained insufficient negative correlations between output and the spread, and output and net outflows. If anything our use of extreme value shocks proved more successful in replicating the properties of the data than the original ones.

To assess the role of commitment, we look at the two left panels of Table 4. The first row shows a comparison of the extent of borrowing and default in the Chatterjee and Eyigungor (2012) when there is and where there is not commitment. The relevant comparison is between the values of the third and seventh columns, and between the values of the fourth and eighth columns (in bold). We see that, unlike with short-term debt, we have a larger mean spread when there is commitment, indicating more default. How can this be? Because the sovereign gets borrows more under commitment yielding more default.<sup>10</sup> So in this model with long-term debt commitment to not default but not to the amount of

<sup>10</sup> Note that these are averages from the stationary distribution (500,000 periods after forgetting initial conditions, and



$\lambda$	$\alpha$	Commitment to Default Circumstances				No Commitment				Commitment to Default Circums. and No Dilution			
		$\mu_r$	$b/y$	$\sigma_r$	$\rho_{r,y}$	$\mu_r$	$b/y$	$\sigma_r$	$\rho_{r,y}$	$\mu_r$	$b/y$	$\sigma_r$	$\rho_{r,y}$
0.05	0.0033	<b>0.088</b>	<b>0.73</b>	0.071	-0.66	<b>0.082</b>	<b>0.70</b>	0.047	-0.75	$\approx 0$	<b>2.9</b>	$\approx 0$	-0.65
0.125	0.0033	0.039	0.85	0.031	-0.73	0.039	0.72	0.023	-0.82	—	—	—	—
0.25	0.0033	0.008	1.09	0.019	-0.60	0.023	0.74	0.015	-0.82	—	—	—	—
0.50	0.0033	$\approx 0$	1.97	$\approx 0$	-0.54	0.015	0.75	0.012	-0.78	—	—	—	—
Annual Model		0.005	1.16	0.011	-0.67	0.043	1.16	0.015	-0.66	—	—	—	—
0.05	0.0040	0.188	0.48	0.150	-0.74	0.163	0.45	0.102	-0.79	—	—	—	—

**Table 4: A Comparison of Chatterjee and Eyigungor (2012) with and without Commitment**

The values of debt over output are per quarter.

dilution does not change very much the properties of the allocation of the standard model.

What about commitment to both the circumstances of default and the amount of dilution (that as we said is equivalent to short-term debt with commitment)? The last panel of the first row gives us the answer. Default becomes essentially zero (about  $10^{-10}$ ) and the sovereign holds a lot more debt, more than triple with commitment limited to the circumstances of default. Clearly, commitment in this case is very powerful.

To investigate what matters further we conduct a few more experiments. Rows two to four of Table 4 display the properties of economies that differ in the length of the debt (lower values of  $\lambda$  correspond to higher debt maturities). Note that the shorter the maturity the lower default and the higher the debt for both commitment and no-commitment, but more drastically for the commitment economies so much so that for a debt of average duration of six months default almost disappears and average debt is more than twice the debt that can be achieved with a five year maturity. The last row of Table 4 re-calibrates the model to annual periods and now commitment to default circumstances alone makes default occur less than 1% of the time without changing the maturity of the debt, indicating that what matters is the amount of time over which the sovereign commits to not default.

An important issue that normally goes unnoticed is the role of the location parameter of the extreme value shocks. When this value is zero (and the implied mean of the maximum of two Gumbel random variables is  $\alpha(\gamma + \log 2)$ ) exclusion from borrowing and lending implicitly excludes the sovereign from the utility associated to these Gumbel random variables, and the costs of defaulting become much larger. This shows up dramatically when we let this consideration affect the cost of defaulting. Letting this consideration shape the answer under commitment changes the outcomes. In particular, instead of having average default of 8.8% and average debt of 73% of quarterly output, we would get average

excluding the first 20 quarters after default) not the choices given the same state.

default of 2.1% and average debt of 2.3 times quarterly output. We take this as a message of caution about how to deal with the extreme value shocks: are they structural objects so that defaulting implies that losing them something fundamental is lost or are they a smoothing device that should not affect average utility. The answer to this question tells how to proceed.

One more point that we want to emphasize is the important value of the variance of the shock. The last row of the Table displays the values under both commitment and non-commitment of the [Chatterjee and Eyigungor \(2012\)](#) with a dispersion parameter  $\alpha$  of .004 vs the value in the original of .0033, an increase of 21% of the variance or 10% of the standard deviation. Here default doubles and average debt shrinks by a third in both economies. The reason is that the arbitrariness of the extreme value shocks effect on the decision grows enough to want to default sometimes regardless of whether commitment is available, sending a cautionary message to the use of extreme value shocks as smoothing devices.

To summarize, commitment understood as the ability to specify one quarter ahead the circumstances of its default is not very powerful, leading, in fact to more default and more debt. Interpreted instead to specify the circumstances of the default of its debt and a guarantee against dilution in the form of a price commitment effectively allows for a much more powerful role, effectively eliminating default.

## 6. Conclusion

In this paper we have explored the role of lack of commitment to the circumstances of default in the sovereign debt environment. We have characterized theoretically the problem under commitment and we have shown how different the properties of the solution are than those of the environment without commitment. The functional equations are completely different.

We have also explored the quantitative properties of the solution for short and for long-term debt and we find that under short-term debt the lack of commitment is absolutely central to the existence of default. If sovereigns had to comply with the fact that default is only feasible when it is specified in the terms of the loans, they would almost always never choose to default. Operationally, this means that default never happens in the economy parameterized à la [Arellano \(2008\)](#), not even if all punishment beyond the inability to save in the period of default is abandoned. The economy in this case could reach a level of debt arbitrarily close to the natural borrowing limit albeit with very low probability. In these cases, the economy collapses to that where the default option is not present. Default is possible, however if a bad enough disaster is in the horizon as our example shows.

With long term debt the quantitative properties are more nuanced. Committing to not defaulting next quarter with debt maturity of 5 years barely changes the outcomes of the non-commitment environment. However, either commitment to not defaulting and to the amount of dilution or committing to not defaulting for a little over a year effectively eliminates default, indicating that the exact meaning of commitment is less important than the horizon over it applies.

We conclude that lack of commitment, and not the coarseness of the debt instruments is the main rationale for the existence of default. Having a default option is not relevant in itself, but only when there is also lack of commitment to the circumstances under which it will be chosen.

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